Edexcel GCE Core Mathematics (C2)

Required Knowledge Information Sheet

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C2 Formulae Given in Mathematical Formulae and Statistical Tables Booklet

- Cosine Rule
  - $a^2 = b^2 + c^2 - 2bc \text{ cosine } (A)$

- Binomial Series
  - $(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^{b2} + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n$
    - where $(n \in \mathbb{N})$
    - and $\binom{n}{r} = {}^n c_r = \frac{n!}{r!(n-r)!}$
  - $(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{1 \text{ x } 2} + \cdots + \frac{n(n-1)\ldots(n-r+1)x^r}{1 \text{ x } 2 \text{ x } \ldots \text{ x } r} + \ldots$
    - $(|x| < 1, n \in \mathbb{R})$

- Logarithms and Exponentials
  - $log_a\, x = \frac{log_b\, x}{log_b\, a}$

- Geometric Series
  - $u_n = ar^{n-1}$
  - $S_n = \frac{a(1-r^n)}{1-r}$
  - $S_\infty = \frac{a}{1-r}$ for $|r| < 1$

- Numerical Integration
  - $\int_a^b y\ dx \approx \frac{1}{2}h\left\{(y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n_1})\right\}, where\ h = \frac{b-a}{n}$

Algebra and Functions

- If *f(x)* is a polynomial and *f(a) = 0*, then *(x - a)* is a factor of *f(x)*
- If *f(x)* is a polynomial and f$\left(\frac{b}{a}\right)$ = 0, then *(ax – b)* is a factor of *f(x)*
- If a polynomial *f(x)* is divided by *(ax – b)* then the remainder is f$\left(\frac{b}{a}\right)$

The Sine and Cosine Rule

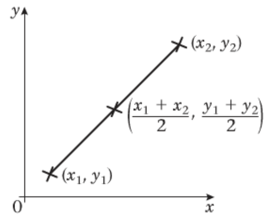
- The sine rule is:
  - $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
  - $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- You can use the sine rule to find an unknown side in a triangle if you know two angles and the length of one of their opposite sides
- You can use the sine rule to find an unknown angle in a triangle if you know the lengths of two sides and one of their opposite angles
- The cosine rule is:
  - $a^2 = b^2 + c^2 – 2bc \cos (A)$
  - $b^2 = a^2 + c^2 – 2ac \cos (B)$
  - $c^2 = a^2 + b^2 – 2ab \cos (C)$
- You can use the cosine rule to find an unknown side in a triangle if you know the lengths of two sides and the angle between them
- You can use the cosine rule to find an unknown angle if you know the lengths of all three sides
- The rearranged form of the cosine rule used to find an unknown angle is:
  - $cos\, A = \frac{b^2+c^2-a^2}{2bc}$
  - $\cos B = \frac{a^2+c^2-b^2}{2ac}$
  - $\cos C = \frac{a^2+b-c^2}{2ab}$
- You can find the area of a triangle using the formula
  - $Area = \frac{1}{2}ab \sin C$
    - If you know the length of two sides (a and b) and the value of the angle C between them

Exponentials and Logarithms

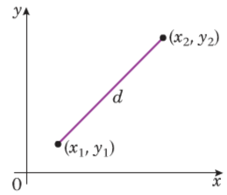
- A function $y = a^x$, or $f(x) = a^x$, *where* a is a constant, is called an exponential function
- $\log_a n = x$ means that $a^x = n$, where a is called the base of the logarithm
- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_{10} x$ is sometimes written as $\log x$
- The laws of logarithms are
    - $\log_a xy = \log_a x + \log_a y$ *(the multiplication law)*
    - $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$ *(the division law)*
    - $\log_a (x)^k = k \log_a x$ *(the power law)*
- From the power law,
    - $\log_a \left(\frac{1}{x}\right) = - \log_a x$
- You can solve an equation such as $a^x = b$ by first taking logarithms (to base 10) of each side
- The change of base rule for logarithms can be written as:
    - $\log_a x = \frac{\log_b x}{\log_b a}$
- From the change of base rule:
    - $\log_a b = \frac{1}{\log_b a}$

Coordinate Geometry in the *(x, y)* Plane

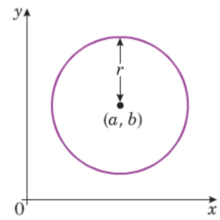
- The mid-point of (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$



- The distance *d* between (x_1, y_1) and (x_2, y_2) is
  d= $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

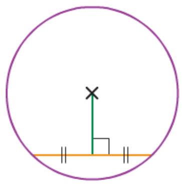


- The equation of the circle centre *(a, b)* radius *r* is
  $(x - a)^2 + (y - b)^2 = r^2$

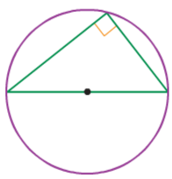


- A chord is a line that joins two points on the circumference of a circle

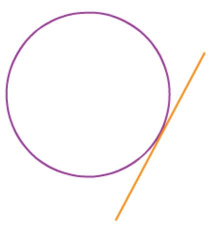
- The perpendicular from the centre of a circle to a chord bisects the chord



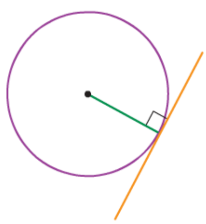
- The angle in a semi-circle is a right angle



- A tangent is a line that meets the circle at one point only



- The angle between a tangent and a radius is 90°

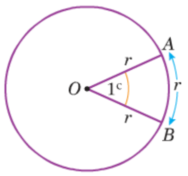


The Binomial Expansion

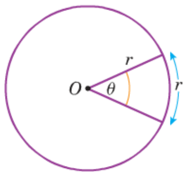
- You can use Pascal's Triangle to multiply out a bracket
- You can use combinations and factorial notation to help you expand binomial expressions. For larger indices it is quicker than using Pascal's Triangle
- $n! = n \times (n-1) \times (n-2) \times (n-3) \times \ldots \times 3 \times 2 \times 1$
- The number of ways of choosing *r* items from a group of *n* items is written $\binom{n}{r}$ or ${}^{n}c_{r}$
- The binomial expansion is:
    - $(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^{2} + \cdots + \binom{n}{r}a^{n-r}b^{r} + \cdots + b^n$
- $(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{1 \times 2} + \cdots + \frac{n(n-1)\ldots(n-r+1)x^r}{1 \times 2 \times \ldots \times r} + \ldots$

Radian Measure and its Applications

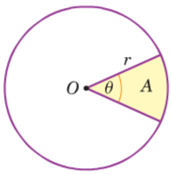
- If the arc AB has length r, then ∠AOB is 1 radian (1^c or 1 rad)



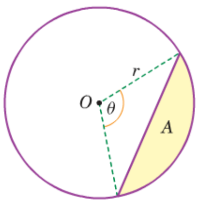
- A radian is the angle subtended at the centre of a circle by an arc whose length is equal to that of the radius of the circle
- $1\ radian = \frac{180°}{\pi}$
- The length of an arc of a circle is $l = r\,\theta$



- The area of a sector is $A = \frac{1}{2}r^2\theta$



- The area of a segment in a circle is $A = \frac{1}{2}r^2(\theta - \sin\theta)$

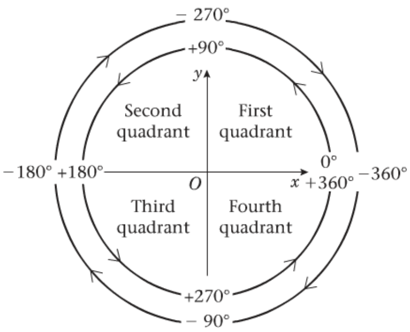


Geometric Sequences and Series

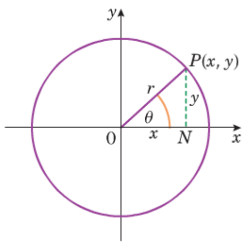
- In a geometric series you get from one term to the next by multiplying by a constant called the common ratio
- The formula for the n^{th} *term = ar* $^{n-1}$ where a = the first term and r = the common ratio
- The formula for the sum to *n* terms is
  - $S_n = \frac{a(1-r^n)}{1-r}$ or,
  - $S_n = \frac{a(r^n-1)}{r-1}$
- The sum to infinity exists if $|r| < 1$ and is $S_\infty = \frac{a}{1-r}$

Graphs of Trigonometric Functions

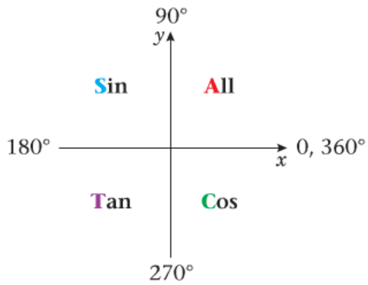
- The *x-y* plane is divided into quadrants



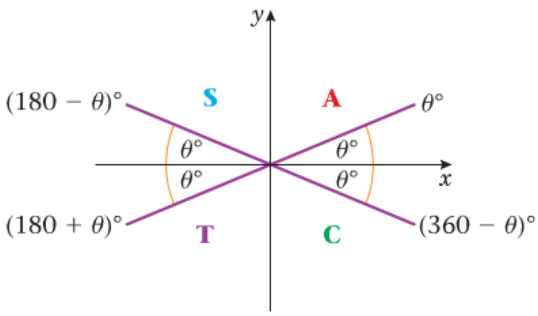
- For all values of Θ, the definitions of Sin (Θ), Cos (Θ) and Tan (Θ) are taken to be ... where x and y are the coordinates of P and r is the radius of the circle
  - $\sin\theta = \frac{y}{r}$
  - $\cos\theta = \frac{x}{r}$
  - $\tan\theta = \frac{y}{x}$



- A cast diagram tell you which angles are positive or negative for Sine, Cosine and Tangent trigonometric functions:
  - In the first quadrant, where Θ is acute, All trigonometric functions are positive
  - In the second quadrant, where Θ is obtuse, only sine is positive
  - In the third quadrant, where Θ is reflex, $180^o < \Theta < 270^o$, only tangent is positive
  - In the fourth quadrant where Θ is reflex, $270^o < \Theta < 360^o$, only cosine is positive
  -



- The trigonometric ratios of angles equally inclined to the horizontal are related :
  - $\text{Sin}\ (180 - \Theta)^o = \text{Sin}\ \Theta^o$
  - $\text{Sin}\ (180 + \Theta)^o = -\ \text{Sin}\ \Theta^o$
  - $\text{Sin}\ (360 - \Theta)^o = -\ \text{Sin}\ \Theta^o$
  - $\text{Cos}\ (180 - \Theta)^o = -\ \text{Cos}\ \Theta^o$
  - $\text{Cos}\ (180 + \Theta)^o = -\ \text{Cos}\ \Theta^o$
  - $\text{Cos}\ (360 - \Theta)^o = \text{Cos}\ \Theta^o$
  - $\text{Tan}\ (180 - \Theta)^o = -\ \text{Tan}\ \Theta^o$
  - $\text{Tan}\ (180 + \Theta)^o = \text{Tan}\ \Theta^o$
  - $\text{Tan}\ (360 - \Theta)^o = -\ \text{Tan}\ \Theta^o$



- The trigonometric ratios of 30^o, 45^o and 60^o have exact forms, given below:

	Sine (Θ)	Cosine (Θ)	Tangent (Θ)
30^o	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45^o	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60^o	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

- The sine and cosine functions have a period of 360^o, (or 2π radians). Periodic properties are :
  - *Sin* $(\Theta \pm 360^o) = \text{Sin } \Theta$
  - *Cos* $(\Theta \pm 360^o) = \text{Cos } \Theta$
- The tangent function has a period of 180^o, (or π radians). Periodic property is:
  - *Tan* $(\Theta \pm 180^o) = Tan\ \Theta$
- Other useful properties are
  - *Sin* $(-\Theta) = -Sin\ \Theta$
  - *Cos* $(-\Theta) = Cos\ \Theta$
  - *Tan* $(-\Theta) = -Tan\ \Theta$
  - *Sin* $(90^o - \Theta) = Cos\ \Theta$
  - *Cos* $(90^o - \Theta) = Sin\ \Theta$

Differentiation

- For an increasing function *f(x)* in the interval (a, b), *f'(x) > 0* in the interval $a \le x \le b$
- For a decreasing function *f(x)* in the interval (a, b), *f'(x) < 0* in the interval $a \le x \le b$
- The points where *f(x)* stops increasing and begins to decrease are called maximum points
- The points where *f(x)* stops decreasing and begins to increase are called minimum points
- A point of inflection is a point where the gradient is at a maximum or minimum value in the neighbourhood of the point
- A stationary point is a point of zero gradient. It may be a maximum, a minimum or a point of inflection
- To find the coordinates of a stationary point:
  - find $\frac{dy}{dx}$ (The gradient function)
  - Solve the equation *f'(x) = 0* to find the value, or values, of x
  - Substitute into *y = f(x)* to find the corresponding values of y
- The stationary value of a function is the value of y at the stationary point. You can sometimes use this to find the range of a function
- You may determine the nature of a stationary point by using the second derivative
  - If $\frac{dy}{dx}$ = 0 and $\frac{d^2y}{dx^2}$ > 0, the point is a minimum point
  - If $\frac{dy}{dx}$ = 0 and $\frac{d^2y}{dx^2}$ <0, the point is a maximum point
  - If $\frac{dy}{dx}$ = 0 and $\frac{d^2y}{dx^2}$ = 0, the point is either a maximum, minimum, or point of inflection
  - If $\frac{dy}{dx}$ = 0 and $\frac{d^2y}{dx^2}$ = 0, but $\frac{d^3y}{dx^3} \neq 0$, then the point is a point of inflection
- In problems where you need to find the maximum or minimum value of a variable y, first establish a formula for y in terms of x, then differentiate and put the derived function equal to zero to then find x and then y

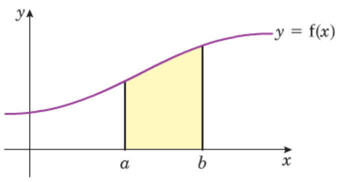
Trigonometrical Identities and Simple Equations

- $Tan\ \theta = \frac{Sin\ \theta}{Cos\ \theta}$ (providing Cos Θ ≠ 0, when Tan Θ is not defined)
- $Sin^2\theta + Cos^2\theta = 1$
- A first solution of the equation *Sin x = k* is your calculator value, α = Sin^{-1} k. A second solution is (180^o – α), or (π – α) if you are working in radians. Other solutions are found by adding or subtracting multiples of 360^o or 2π radians.
- A first solution of the equation Cos *x = k* is your calculator value, α = Cos^{-1} k. A second solution is (360^o – α), or (2π – α) if you are working in radians. Other solutions are found by adding or subtracting multiples of 360^o or 2π radians.
- A first solution of the equation Tan *x = k* is your calculator value, α = Tan^{-1} k. A second solution is (180^o + α), or (π + α) if you are working in radians. Other solutions are found by adding or subtracting multiples of 180^o or π radians.

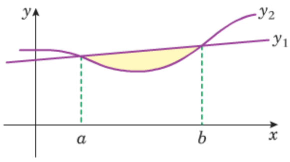
Integration

- The definite integral $\int_a^b f'(x)\,dx = f(b) - f(a)$

- The area beneath a curve with equation *y = f(x)* and between the lines *x =a* and *x = b* is:
    - $Area = \int_a^b f(x)dx$



- The area between a line (equation y_1) and a curve (equation y_2) is given by:
    - $Area = \int_a^b (y1, y2)dx$



- The Trapezium rule is:
    - $\int_a^b y\ dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n_1})\}$
        - $where\ h = \frac{b-a}{n}$ and $y_i = f(a + ih)$