Revision Notes for OCR Core 2

Sine and Cosine Rules

Area of a triangle = ½ x base x height = ½ a b sin C where C is angle between a and b

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ **or even** $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc \cos A$ Can swap letters around but angle A, B or C must be between the lengths you know.

Logarithms

$f(x) = b^x$ where b = base, x = exponent.

$y = b^x$ **and** $x = \log_b y$ If using base 10 then can use calculator but otherwise work out mentally. e.g If $2 = \log_3 y$ then $y = 3^2 = 9$ Or $x = \log_4 64$ then $64 = 4^x$ so x = 3.

log (pq) = log p + log q Split number into factor pairs then use this rule to simplify.
log (p/q) = log p – log q Split number into 2 divisible numbers then use this rule.
log (p^x) = x log p Useful when want x so log both sides then rearrange. Can use calculator as $\log_{10}$

Factors and remainders

If f(x) has (x – 3) is a factor, then f(3) = 0. No remainder.
If f($3x^3 - 2x^2 + x - 18$) and x – 2 is a factor, then $(x-2)(ax^2 + bx + c) = 0$ and compare coefficients to find a, b and c then solve for the quadratic.

If don't know any factors, experiment with numbers to find one then do as above.

If not a factor, then substitute in, what's left is the remainder.
e.g Is (x – 4) a factor of $(3x^3 - 2x^2 + x - 6)$, $f(4) = 3 \times 4^3 - 2 \times 4^2 + 4 - 6 =$ **158** (remainder)
then $(3x^3 + 2x^2 + x - 6) = (x-4)$ (**$\underline{ax^2 + bx + c}$**) + R. Compare coefficients to find a, b and c.
This expression is the **quotient**.

Sequences

If goes up or down in steps of d = **Arithmetic**
If goes up by a common multiplier, e.g x 2, x 3 etc then **Geometric**

Arithmetic

$U_r = a + (r-1)d$ where d is common difference, a = first term, r = term number.
Can add common difference or subtract to move up or down the sequence.
L = a + (n-1)d **sum S = ½ n (a + L)** **or** **S = ½ n (2a + (n – 1)d)**

Geometric

Has a common multiplier called common ratio.
$U_{i+1} = r U_i$ where r = common ratio (2nd term / 1st term or any other pairing) (term to term formula)
$U_i = a r^{i-1}$ where a = first term. (position to term formula)
Could use 4th term then i becomes 4 less as new starting point. e.g If r = 2 and $U_4 = 8$, then 8th term $U_8 = 8 \times 2^{8-4-1} = 8 \times 2^3 = 64$.
Could find a by working backwards then use formula as normal.

Sum $S_n = \frac{a(1 - r^n)}{1 - r}$ and if n tends to infinity, then S_n tends to $\frac{a}{1 - r}$

Binomial Theorem

Used to expand $(x + y)^n$ where n is a positive integer.

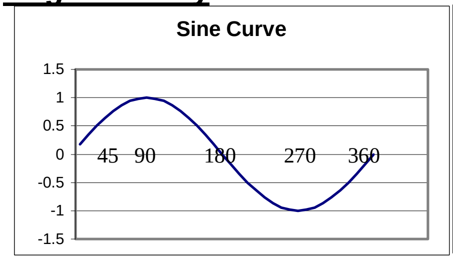
Use Pascal's triangle to give the coefficients and remember that the powers drop for the x's and increase for the y's. (e,g from x^n for the 1st term to $x^{n-1}y$ for the 2nd etc to end with y^n.) Easiest to write out sequence using x and y's then substitute in the actual terms afterwards remembering to apply the power to the whole expression.

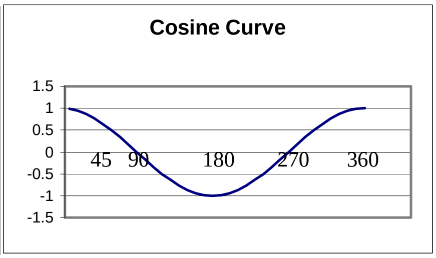
$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 = \ldots + \binom{n}{n}y^n$ where $\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$

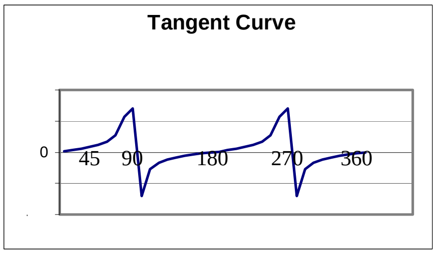
(If want rth term of the nth row of Pascal's triangle use ${}^nC_r$)

Use the formula booklet for full formula and just substitute in numbers.

Trigonometry







Sine

Repeats every 360º. Supplementary angle at **180º - θ.**

Cosine

Repeats every 360º. Supplementary angle at **360º - θ**

Tangent

Repeats every 180º.

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad$ **$\sin^2\theta + \cos^2\theta = 1$**

If finding sin 2θ then let 2θ = α and find solution for α then ½ your answers. Remember that range of possible values for α is twice as big (e.g. $0 < \theta < 360º$ becomes $0 < \alpha < 720º$) as will halve the solutions as $\theta = \alpha \div 2$.

Radians

$2\pi = 360º$ $\quad$ Arc Length **$s = r\theta$** Area of sector **$A = \frac{1}{2}r^2\theta$** (Must be in radians)

$\pi = 180º$

Often better to calculate angle in degrees then change to a fraction of π especially when using trigonometry.

Integration

Opposite of differentiation.

$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c$ $\quad$ If there is a coefficient in front of the x, then it just stays their and multiples the answer. C = arbitrary constant.

If you know a point on the curve, then can integrate the gradient formula f '(x) to find the formula for the function and use coordinates to find the constant c.

Integrating a function also finds the area beneath the curve between 2 points and is a definite integral. If the limits are x=3 and x = 1 then integrate with respect to x and substitute in the 2 x values (3 and 1) and subtract.

If 2 curves and want area between them, can find the areas under each separately then subtract or subtract the equations first, then integrate and substitute in.

Trapezium Rule

Used if you cannot integrate the function. Split curve up into separate trapeziums of equal width and find approximate area for each. Number of intervals (separate trapeziums will be given.) Area of a trapezium **$= \frac{1}{2}h(y_0 + y_1)$** where $h = x_1 - x_0$.

Can use the formula area = **$\frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$** where $h = \frac{x_n - x_0}{n}$ and n = number of intervals. (x_n and x_0 are the upper and lower limits).