Integration & Differentiation

What you are given and what you need to know in C4

FORMULAE FOR EDEXCEL

2013/14

Integration & Differentiation

What you are given and what you need to know in C4

Recap of C3 facts

Exact Values of trigonometric functions

x° (deg)	x° (rad)	sin	cos	tan
0	0	0	1	0
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	$\frac{\pi}{2}$	1	0	-
180	π	0	-1	0

Rules and facts

1. $\text{Sin}^2x + \cos^2x = 1$
2. $\text{Tan } x = \frac{\sin x}{\cos x}$
3. $\text{Cosec } x = \frac{1}{\sin x}$
4. $\text{Sec } x = \frac{1}{\cos x}$
5. $\text{Cot } x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$

Applying these rules

Dividing (1) by $\sin^2x$ will give you: $1 + \cot^2x = \text{cosec}^2x$

Dividing (1) by $\cos^2x$ will give you: $\tan^2x + 1 = \sec^2x$

(*) means the rule is given in the Edexcel Formula book

Differentiation

Parametric Equations

If y = f(t) and x = g(t), then:

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{\mathrm{dy}}{\mathrm{dt}} \div \frac{\mathrm{dt}}{\mathrm{dx}}$$

Implicit Differentiation

When f(x,y) = g(x,y), differentiate implicitly: that is differentiate w.r.t. y and include dy/dx . The solution can simplified where necessary.

Example: $y^2 = xy + x + 2$

(Hint: Use the product rule for xy)

$$2y\frac{dy}{dx} = x \times 1 \times \frac{dy}{dx} + y \times 1 + 1$$

a^x

$$\frac{d(a^x)}{dx} = a^x \ln(a)$$

Proof of a^x

Start with $y = a^x$

Take logs of both sides $\ln(y) = \ln(a^x)$

$\ln(y) = x\ln(a)$

Differentiate implicitly $\frac{1}{y} \times \frac{dy}{dx} = \ln(a)$

Rearrange and substitute for y

$$\frac{d(a^x)}{dx} = a^x \ln(a)$$

(*) means the rule is given n the Edexcel Formula book

Integration

Rules for Integration

Integration by substitution

There is no simple rule for integration by substitution, you must follow these steps:

- You'll be given an integral which is made up of two functions of x.

$$\int 4xe^{(x^2-1)}dx$$

- Substitute *u* for one of the functions of x to give function which is easier to integrate.

$$Choose\ u = x^2 - 1, to\ give\ e^u$$

- Next, find $\frac{du}{dx}$, and rewrite it so that dx is on its own.

$$\frac{du}{dx} = 2x, so\ \ xdx = \frac{1}{2}du$$

- Rewrite the original integral in terms of *u* and *du*.

$$Substituting\ in\ for\ xdx\colon\ \int 4\,e^u\,xdx = \int 2e^u\,du$$

- Integrate and substitute back for *u* at the end.

$$2\int e^u du = 2e^u + c = 2e^{(x^2-1)} + c$$

Integration by parts*

When u=f(x) and v=g(x), then:

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

Choose your u and v functions carefully to make the integral easier.

(*) means the rule is given in the Edexcel Formula book

Volume of revolution: Cartesian

$$V = \pi \int_{x_1}^{x_2} y^2 \, dx$$

This describes the volume generated when the curve of y = f(x) from x_1 to x_2 is rotated 360° about the x-axis.

Volume of revolution: Parametric

$$V_x = \pi \int_a^b y^2 \frac{dx}{dt} \, dt$$

This describes the volume generated when the curve is defined by its parametric form (x(t), y(t)) in the interval (a,b) is rotated 360° about the x-axis.

Both equations for the volumes of revolution can be adjusted for rotation about the y-axis by substituting x for y and vice versa.

(*) means the rule is given n the Edexcel Formula book

Standard Integrals you should know:

$$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c \qquad \text{where } n \neq 1$$

Exponential functions

$$\int e^x \, dx = e^x + c$$

$$\int e^{ax+b} \, dx = \frac{1}{a} e^{ax+b} + c$$

Other functions

$$\int \frac{1}{x} dx = ln|x| + c$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} ln|ax+b| + c$$

$$\int \frac{f'(x)}{f(x)} dx = ln|f(x)| + c$$

This rule leads to these standard integrals (*) :

$$\int cosec(x) \, dx = -ln|cosec(x) + \cot(x)| + c$$

$$\int sec(x) \, dx = ln|sec(x) + \tan(x)| + c$$

$$\int cot(x) \, dx = ln|sin(x)| + c$$

(*) means the rule is given in the Edexcel Formula book

Using functions and derivatives

$$\int \frac{du}{dx} f(u)\, dx = f(u) + c$$

$$\int (n+1) f'(x)\, [f(x)]^n\, dx = [f(x)]^{n+1} + c$$

Trigonometric Integration

Basics

Learn these facts and do not confuse them with the rules for differentiation.

$$\int \sin(x)\, dx = -\cos(x) + c$$

$$\int \cos(x)\, dx = \sin(x) + c$$

Summary (+ constant)

y=f(x)	$\int \boldsymbol{f(x)dx}$	**In formula book**
Cos x	Sin x	
Sin x	-Cos x	
$\sec^2(kx)$	$\frac{1}{k}$ tan (kx)	*
tan(x)	$ln\lvert\sec(x)\rvert$	*
cot(x)	$ln\lvert\sin(x)\rvert$	*
sec (x)	$ln\lvert\sec(x)+\tan(x)\rvert$	*
cosec(x)	$-ln\lvert\mathrm{cosec}(x)+\cot(x)\rvert$	*

(*) means the rule is given n the Edexcel Formula book

Applying these facts

By the chain rule: $\frac{d[\sin(ax+b)]}{dx} = a\cos(ax + b)$

Hence: $\int \cos(ax + b)\, dx = \frac{1}{a}\sin(ax + b) + c$

It follows that: $\int \sin(ax + b)\, dx = -\frac{1}{a}\cos(ax + b) + c$

By the quotient rule: $\frac{d[\tan(x)]}{dx} = sec^2(x)$

Hence: $\int sec^2(x)\, dx = \tan(x) + c$

Also: $\int sec^2(kx)\, dx = \frac{1}{\mathrm{k}}\tan(kx) + c$ (*)

Thus: $\int sec^2(ax + b)\, dx = \frac{1}{\mathrm{a}}\tan(ax + b) + c$

(*) means the rule is given in the Edexcel Formula book