MEI

Mathematics in Education and Industry

MEI Structured Mathematics

Module Summary Sheets

Mechanics 1

(Version B: Reference to new books)

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Summary M1 Topic 1: **Motion**

References:
Chapter 1
Pages 1-4

Exercise 1A
Q. 3

Terminology

Displacement s: distance in a certain direction
Distance is the magnitude of the displacement
Velocity v: rate of change of displacement
Speed is the magnitude of the velocity
Acceleration a: rate of change of velocity
Retardation (deceleration) is –ve acceleration

A **scalar** is a quantity that has magnitude only
A **vector** is a quantity that has magnitude and direction.

vector	scalar
displacement	distance
velocity	speed
acceleration	

References:
Chapter 1
Pages 5-8

Exercise 1B
Q. 3

References:
Chapter 1
Pages 10-11

Exercise 1C
Q. 5

Graphs

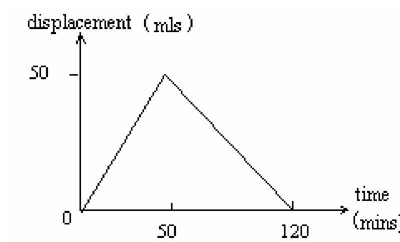
Time is plotted on the horizonatal axis.
Displacement– time graph
The velocity at a point is the gradient of the curve.

Velocity-time graph
Acceleration at a point is the gradient of the graph at that point.

$$\text{Average Velocity} = \frac{\text{total displacement}}{\text{total time}}$$

$$\text{Average Acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

E.g. A car travelling at constant speed goes up a motorway for 50 miles, turns round and immediately travels back. The first part takes 50 mins and the second part takes 70 mins.



After 2 hours:
Displacement = 0 Distance = 100 miles
Average Vel = 0 Average Speed = 50 mph
Velocity for first part = 60 mph
Velocity for second part = -43 mph (to 3 s.f.)

References:
Chapter 1
Pages 12-15

Exercise 1D
Q. 6

Areas under graphs

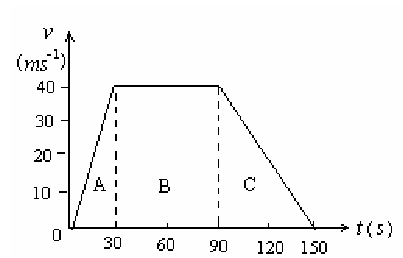
The area between a speed-time graph and the x- axis represents the distance travelled.

The area between a velocity-time graph and the x- axis represents the displacement. An area below the axis is taken as negative.

When the velocity (or speed) is modelled by constant acceleration then the sections of the velocity-time graph will be straight lines. The area under the graph will therefore be a triangle, trapezium or rectangle and can therefore be calculated easily be elementary mensuration.

If the graph is a curve (i.e. not constant acceleration) then the area can be found by integration or estimated by numerical approximation (see C 2).

E.g. the graph represents the motion of a train between 2 stations.
(i) Find the acceleration for each part of the journey.
(ii) How far apart are the two stations?



(i) For Section A: Acceleration $= \frac{\text{Velocity}}{\text{Time}} = \frac{40}{30} = \frac{4}{3}\,\text{m s}^{-2}$

For section B:Acceleration = 0

For Section C:acceleration $= \frac{\text{Velocity}}{\text{Time}} = \frac{-40}{60} = -\frac{2}{3}\,\text{m s}^{-2}$

(ii) Area $A = \frac{1}{2}.30.40 = 600$

Area $B = 60.40 = 2400$

Area $C = \frac{1}{2}60.40 = 1200$

Total = 4200 metres.

Competence statements k1, k2, k3, k4

References:
Chapter 2
Pages 20-25

Example 2.1
Page 24

Exercise 2A
Q. 7

Constant acceleration formulae

$v = u + at$

$v^2 = u^2 + 2as$

$s = ut + \frac{1}{2}at^2$

$s = vt - \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

s = displacement
t = time
u = initial velocity
v = final velocity
a = acceleration

When velocity is constant $s = ut$

- Always define the positive direction
- Units must be consistent
- Equations are for constant acceleration

E.g. A body moves from rest in a straight line with an acceleration of 2 ms^{-2}. Find its displacement after 4 sec.

$u = 0,\ a = 2,\ t = 4$

Using $s = ut + \frac{1}{2}at^2$

$\Rightarrow s = 0 + \frac{1}{2}\times 2 \times 4^2 = 16$ i.e. the displacement is 16 metres.

E.g. A particle is hit across ice with an initial velocity of 10 m s^{-1}. If the retardation is 0.6 m s^{-2} find how long it takes to stop and how far it has travelled.

$u = 10,\ v = 0,\ a = -0.6$

Using $v = u + at \Rightarrow 0 = 10 - 0.6t \Rightarrow t = \frac{50}{3}$

Using $s = ut + \frac{1}{2}at^2$

$\Rightarrow s = 10\frac{50}{3} - \frac{1}{2}\times 0.6 \times \left(\frac{50}{3}\right)^2 = \frac{250}{3}$

i.e. the time is $16\frac{2}{3}$ sec and the displacement is $83\frac{1}{3}$ metres

References:
Chapter 2
Page 26

Exercise 2A
Q. 3

Vertical motion due to gravity

In free fall under gravity a body will fall towards the centre of the earth This may be modelled by a constant acceleration *g* (approximately 9.8ms^{-2})

If a body is thrown upwards

- At highest point $v = 0$
- v is negative on the way down
- $a = -g$
- Motion up and down is symmetrical until it returns to its starting point
- Displacement is negative below the point of projection

E.g. A stone is thrown upwards from the top of a tower of height 40 m with a speed of 14 m s^{-1}. Find the greatest height and the time taken to reach the ground.

Taking the origin to be where the the stone is thrown:

With $u = 14,\ v = 0,\ a = -9.8$ Using $v^2 = u^2 + 2as$

$\Rightarrow 0 = 196 - 2\times 9.8 \times s$

$\Rightarrow s = 10;$ Greatest ht $= 10 + 40 = 50$ metres.

When s= -40 , using $s = ut + \frac{1}{2}at^2$

$\Rightarrow -40 = 14t - \frac{1}{2}\times 9.8 \times t^2$

$\Rightarrow 4.9t^2 - 14t - 40 = 0$

Solving for t by the quadratic formula and taking +ve root as time $> 0 \Rightarrow t \approx 4.62 \Rightarrow$ the time is 4.62 secs.

References:
Chapter 2
Pages 30,31

Exercise 2B
Q. 3, 4

Non-zero displacement

The formulae above assume $s = 0$ when $t = 0$.

If the particle is not at the origin but at $s = s_0$ when $t = 0$, the formula $s = ut + \frac{1}{2}at^2$ becomes $s - s_0 = ut + \frac{1}{2}at^2$.

You need to replace s by $s - s_0$ in every equation.

E.g. Example above, taking the origin to be the ground.

With $u = 14,\ v = 0,\ a = -9.8$ Using $v^2 = u^2 + 2a(s - s_0)$

$\Rightarrow 0 = 196 - 2\times 9.8 \times (s - 40)$

$\Rightarrow s = 40 + \frac{196}{2\times 9.8} = 40 + 10 = 50$

$\Rightarrow$ Distance = 50 metres. (as above)

When $s = 0$, using $s = 40 + ut + \frac{1}{2}at^2$

$\Rightarrow 0 = 40 + 14t - \frac{1}{2}\times 9.8 \times t^2$

$\Rightarrow 4.9t^2 - 14t - 40 = 0 \Rightarrow t \approx 4.62$ taking +ve root

$\Rightarrow$ the time is 4.62 secs (as above)

Summary M1 Topic 3: ***Force and Newton's Laws of motion –1***

References:
Chapter 3
Pages 37-38

Forces

A force causes a change in motion unless it is balanced by an equal and opposite force.
The S.I. unit is the newton.
Force is a vector.

Force diagrams

Drawing a force diagram is a crucial stage of problem solving. It makes you consider the detail of the problem, what forces are acting and what state the body is in.

- The diagram should be large, clear and should contain all the information of the problem.
- Always include acceleration where appropriate.
- Take care to ensure that all the forces acting are included, and are along the line of action.
- Show forces on individual bodies.
- When more than one body is involved, consider each body separately.
- Force lines should have arrows and labels.

References:
Chapter 3
Page 39

Exercise 3A
Q. 4

Centre of mass and the particle model

The centre of mass of a body is a point through which the total weight acts. It is often easier to model a body (which has size) by a single weight acting at a point. The body is then considered to behave like a point mass.

References:
Chapter 3
Pages 39-40
Pages 47-50

Newton's third law

When one object exerts a force on another there is always a reaction which is equal in magnitude and opposite in direction to the acting force.

References:
Chapter 3
Pages 41-42

Exercise 3B
Q. 1

Friction

Friction is a resistive force that acts to oppose sliding between surfaces in contact.
The model is often simplified in this module by assuming that there is no friction. In this case the surfaces in contact are said to be "smooth".

References:
Chapter 3
Page 41

Exercise 3B
Q. 2

Normal Reaction

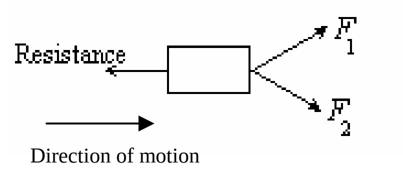
When one surface is prevented from moving into the space occupied by another surface, then a force is exerted at right angles to the surfaces. This is called the Normal Reaction.

Newton's first law

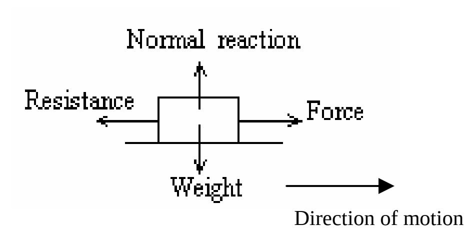
Every body continues in a state of rest or uniform motion in a straight line unless acted on by a resultant external force.

Types of forces

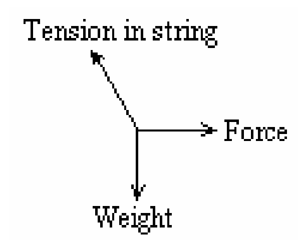
E.g. a sledge being pulled by two people.



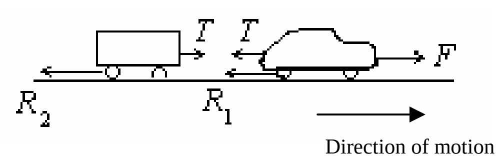
E.g. an object being pulled along a table.



E.g. Mass on the end of a string being pulled to one side.

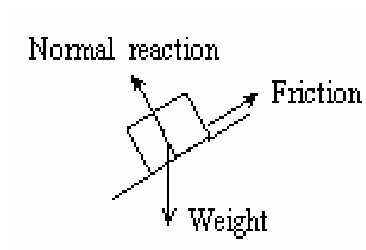


E. g. Car pulling a trailer



(T is the tension in the couplings, F is the driving force and R_1 and R_2 the resistances to motion.)

E.g. a stationary body on a slope.



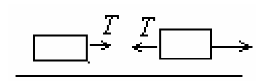
Mechanics 1
Version B: page 4
Competence statements d1, d2, n1

References:
Chapter 3
Pages 47-48

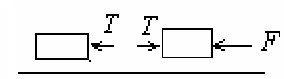
Tension and Thrust

When a force has the effect of pulling then there is **tension** in the connection.
When a force has the effect of pushing then there is **thrust** in the connection. The connection is said to be in compression.
A string can only be in tension and that tension has its direction along the line of the string.

E.g. When an engine pulls a truck then there is tension is the connection, but if the engine pushes then there is thrust.



Tension



Thrust

References:
Chapter 3
Pages 48-49

Exercise 3D
Q. 6

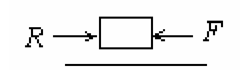
Resultants

The resultant of two vectors is the combined effect of those vectors.
This can be applied to all vectors: Force, velocity, displacement, etc.
A resultant force is the single force which could replace a system of forces.

Equilibrium

When a body is in equilibrium, (at rest or moving with constant velocity) the forces on it balance.
i.e. the resultant force in any direction is zero.

E.g. A train with a driving force of F experiencing resistance of R.



If the two forces are equal and opposite then the train will either remain at rest or move at constant speed.

If $F > R$ then there will be an acceleration in the direction of F.

References:
Chapter 3
Pages 49-50

Exercise 3C
Q. 6

Newton's second Law

Acceleration is proportional to force.

$$F = ma$$

The unit of force is the newton. A force of 1 newton will give a mass of 1 kg an acceleration of 1 m s^{-2}.

E.g. An overall force of 20 N acting on a body with mass 10 kg produces an acceleration of a m s^{-2}.

Using $F = ma$ gives $20 = 10a \Rightarrow a = 2\,\text{m s}^{-2}$

If the body is initially at rest then $v = at$ and $s = \frac{1}{2}at^2$.

So after 3secs the velocity in the direction of the force is $6\,\text{m s}^{-1}$ and the displacement is $9\,\text{m}$.

References:
Chapter 3
Pages 50-51

Weight

The mass of an object is related to the amount of substance. It is a scalar quantity.
The weight is the force of gravity pulling the body towards earth.

$$W = mg$$

Note: Society in general gets the definition muddled (e.g. a bag of potatoes weighs 5 kg, whereas this is actually the mass).

E.g. The weight of 2 kg of apples is 2g Newtons.
If g = 9.8 m s^{-2} then W = 19.6 N.
(g is not always 9.8 m s^{-2} on the surface of the earth but it is often taken to be this value or in exercises the value of 10 m s^{-2} is sometimes used. On the surface of the moon the value of g is very different and so the weight of 2 kg of apples will be different.)

References:
Chapter 3
Pages 52-53

Example 3.6
Page 53

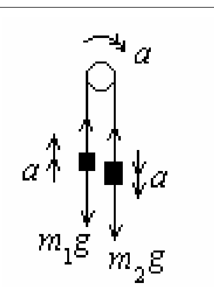
Pulleys

A pulley is used to change the direction of a force.
A pulley is usually modelled as being smooth.
The result of this is that when a string passes over the pulley the tension in the string is the same either side of the pulley.

E.g. If $m_1 = m_2$ then the system is in equilibrium.

If m_1 = 2 kg, m_2 = 3 kg then the system is not in equilibrium. m_2 will accelerate downwards and m_1 upwards.

(The pulley must be smooth and the string light and inextensible.)



Exercise 3D
Q. 4

Mechanics 1
Version B: page 5
Competence statements n1, n2

Summary M1 Topic 4: ***Newton's Laws along a line***

References:
Chapter 4
Pages 58-59

Exercise 4A
Q. 1(i),(v)

Equation of motion

If there is a resultant force on a body then it is not in equilibrium and there will be an acceleration in the direction of the force.

$\mathbf{F} = m\mathbf{a}$ (Newton's 2nd law)

If there is no resultant force on a body then it is in equilibrium. That means that the particle is either at rest or moving with constant velocity.

References:
Chapter 4
Page 62

Exercise 4B
Q. 5

Solving problems

- Draw a large, clear, complete force diagram, showing the direction of any motion
- A body will either be accelerating or in equilibrium. Show acceleration on the force diagram.
- Obtain equations by resolving in the direction of acceleration and using $F = ma$
- Make it clear which object each equation applies to
- Solve the equations.

References:
Chapter 4
Pages 65-67

Exercise 4C
Q. 1, 4

Connected Bodies

Draw a force diagram for each body separately. Obtain equations by considering each body separately. If motion is in the same direction, we can consider the system as one body, provided "internal" forces are not required.

References:
Chapter 4
Page 67

Mathematical Modelling

Mathematical modelling is making assumptions in order to simplify the mathematics. Carrying out experiments will enable you to compare your model to reality.
Examples of modelling:
All forces act through a point
Pulleys are smooth
Strings are light and inextensible
An object is a particle

References:
Chapter 4
Pages 74-75

Air resistance

The assumption that there is no air resistance is an example of mathematical modelling.

Mechanics 1
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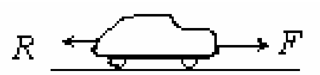
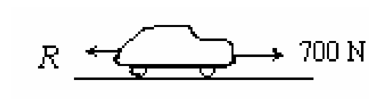
Competence statements n1, n2, n3

E.g. A car of mass 500 kg accelerates at 1.5 m s^{-2}. Resistive forces are 2 N per kg. Find the driving force, F.



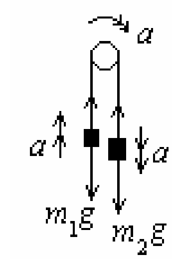
$$\text{Resultant Force} = \text{mass} \times \text{acceleration}$$
$$\Rightarrow F - 500 \times 2 = 500 \times 1.5$$
$$\Rightarrow F = 1000 + 750 = 1750\text{N}$$

E.g. The driver changes the forward force to 700N. What will happen to the car?



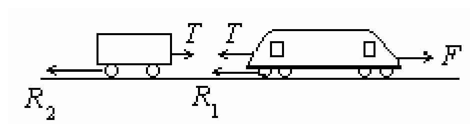
$$700 - 500 \times 2 = 500a$$
$$\Rightarrow 500a = 700 - 1000 = -300$$
$$\Rightarrow a = -0.6 \text{ so the car will decelerate.}$$

E.g. If $m_1 = m_2$ then the system is in equilibrium.
If $m_1 = 2$ kg, $m_2 = 3$ kg then the system is not in equilibrium. m_2 will accelerate downwards and m_1 upwards.



$$m_2 g - T = m_2 a$$
$$T - m_1 g = m_1 a$$

E.g. An engine of mass 20 tonnes is pulling a truck of mass 15 tonnes. Resistances to motion are 1N per kg. If the driving force is 40kN find the acceleration and the tension in the coupling.



For the whole train Total resistances = 35000

$$F = ma \Rightarrow 40000 - 35000 = 35000a$$
$$\Rightarrow a = \frac{5000}{35000} = \frac{1}{7}\text{ms}^{-2}$$
$$\text{For the engine}: 40000 - T - 20000 = 20000 \times \frac{1}{7}$$
$$\Rightarrow T = 40000 - 20000 - \frac{20000}{7} \approx 17143\text{N}$$
$$\text{Or for the truck}: T - 15000 = 15000 \times \frac{1}{7}$$
$$\Rightarrow T \approx 17143\text{N.}$$

(N.B. The tension, T, cannot be found by considering the train as a single body.)

References:
Chapter 5
Pages 78 - 80

Exercise 5A
Q. 4

A **scalar** has magnitude only.
A **vector** has magnitude and direction.

E.g. time, distance, speed, mass are scalars.
E.g. displacement, velocity, acceleration, weight are vectors.

Vector diagrams

Vectors are represented by arrowed lines.
The length represents the magnitude and the arrow the direction.
Parallel vectors are multiples of each other.
Displacement vectors have fixed positions, e.g. position vector.
Free vectors are not fixed by position, e.g. velocity.

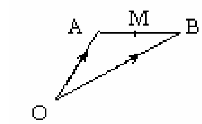
E.g. M is the midpoint of AB where $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. Find $\overrightarrow{OM}$ in terms of **a** and **b**.

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\mathbf{a} + \mathbf{b} = \mathbf{b} - \mathbf{a}$$

$$\Rightarrow \overrightarrow{AM} = \frac{1}{2}\overrightarrow{AB} = \frac{1}{2}(\mathbf{b} - \mathbf{a})$$

$$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$$

$$= \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2}(\mathbf{b} + \mathbf{a})$$



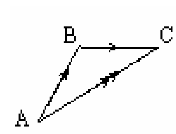
References:
Chapter 5
Page 80

Exercise 5A
Q. 9

Adding and subtracting vectors

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

$$\overrightarrow{AC} - \overrightarrow{BC} = \overrightarrow{AB}$$



References:
Chapter 5
Pages 84-87

Exercise 5B
Q. 6

References:
Chapter 5
Pages 89-91

Exercise 5C
Q. 4

The *x*-*y* plane

The position vector of a point P is given by the vector from the origin $\overrightarrow{OP}$.

Unit Vectors have magnitude 1
i and **j** are used to denote unit vectors in the *x* and *y* directions.

The vector $\overrightarrow{OP}$ can be thought of as a translation from O to P. If P has coordinates (x, y) then

$$\overrightarrow{OP} = x\mathbf{i} + y\mathbf{j} \text{ or } \begin{pmatrix} x \\ y \end{pmatrix}$$

The magnitude of the vector $\overrightarrow{OP}$, $|\text{OP}|$, is the distance from O to P. If P has coordinates (x, y) then

$$|\text{OP}| = \sqrt{x^2 + y^2}$$

The direction of the vector $\overrightarrow{OP}$ is usually taken to be the angle, measured anti-clockwise from the Ox axis.

The direction of $\overrightarrow{OP}$ is $\tan^{-1}\dfrac{y}{x}$.

E.g. $\mathbf{a} = 3\mathbf{i} - 4\mathbf{j}$, $\mathbf{b} = 5\mathbf{i} + \mathbf{j}$
Find (i) $\mathbf{a} + \mathbf{b}$, (ii) $\mathbf{a} - \mathbf{b}$,
(iii) k such that $\mathbf{a} + k\mathbf{b}$ is parallel to the x-axis.

(i) $\mathbf{a} + \mathbf{b} = (3\mathbf{i} - 4\mathbf{j}) + (5\mathbf{i} + \mathbf{j}) = 8\mathbf{i} - 3\mathbf{j}$
(ii) $\mathbf{a} - \mathbf{b} = (3\mathbf{i} - 4\mathbf{j}) - (5\mathbf{i} + \mathbf{j}) = -2\mathbf{i} - 5\mathbf{j}$
(iii) $\mathbf{a} + k\mathbf{b} = (3\mathbf{i} - 4\mathbf{j}) + k(5\mathbf{i} + \mathbf{j})$
$= (3 + 5k)\mathbf{i} - (k - 4)\mathbf{j}$
Parallel to x-axis means zero **j** component
$\Rightarrow k = 4 \Rightarrow \mathbf{a} + k\mathbf{b} = 23\mathbf{i}$

Three dimensions

Results from 2-D are simply extended into 3-D.
A third axis, the *z*-axis, and its corresponding unit vector, **k**, is introduced.
If the point P is at position (x, y, z)

then $$\overrightarrow{OP} = (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

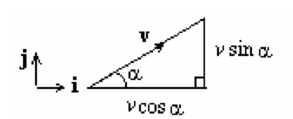
References:
Chapter 5
Pages 93-94

Exercise 5D
Q. 5

Resolving vectors

A vector **v** can be split into components in two (usually) perpendicular directions by resolving it into those directions.

E.g. resolving **v** horizontally and vertically gives
$\mathbf{v} = v\cos\alpha\,\mathbf{i} + v\sin\alpha\,\mathbf{j}$



References:
Chapter 5
Pages 97, 98

Exercise 5E
Q. 7

Velocity triangles

The sum of 2 velocities can be found by drawing a triangle where two sides represent the two vectors in magnitude and direction. The third side represents the resultant in magnitude and direction.
The triangle can be drawn accurately or solved using trigonometry.

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Competence statements v1, v2, v3, v4, v5

Summary M1 Topic 6: ***Projectiles***

References:
Chapter 6
Pages 101-104

Exercise 6A
Q. 1(i),2(i), 3(ii)

References:
Chapter 6
Pages 107-108

Exercise 6B
Q. 2

Exercise 6C
Q. 4

References:
Chapter 6
Page 118

Exercise 6D
Q. 2

References:
Chapter 6
Page 126-128

Exercise 6E
Q. 6

Projectiles

A projectile is a body given an initial velocity which then moves freely under gravity.

Modelling Assumptions:

- no air resistance
- the body is a particle and so there is no spin

Horizontal and vertical components can be analysed separately (or a vector approach may be used), with the positive direction upwards and the origin at the point of projection.

i.e. $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ or $v_y = u_y - gt, \quad v_x = u_x$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ or $y = u_y t - \frac{1}{2}gt^2, \quad x = u_x t$

The acceleration vector is $\begin{pmatrix} 0 \\ -g \end{pmatrix}$ ms^{-2}

The initial velocity vector is $\begin{pmatrix} u_x \\ u_y \end{pmatrix} = \begin{pmatrix} u\cos\alpha \\ u\sin\alpha \end{pmatrix}$ ms^{-1}

where u is the initial speed
and α is the angle of projection.

Properties:

- The path is parabolic
- It is symmetric about a vertical line through its highest point
- The greatest height occurs when $v_y = 0$
- The body returns to the level of projection when $y = 0$

Speed and direction can be found at any point by considering the magnitude and direction of velocity. Position can be found by considering the components x and y.
Constant acceleration formulae are used.

Special results:

Know how to use (but it is not necessary to learn them

Time of flight: $t = \dfrac{2u\sin\alpha}{g}$

Range: $x = \dfrac{u^2 \sin 2\alpha}{g}$: Max. Range: $R = \dfrac{u^2}{g}$ (when $\alpha = 45^o$)

Greatest height: $y = \dfrac{u^2 \sin^2\alpha}{2g}$

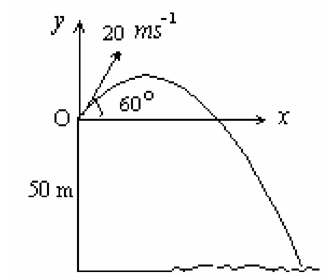
Equation of trajectory: $y = x\tan\alpha - \dfrac{g}{2u^2}x^2\left(1 + \tan^2 x\right)$

Mechanics 1
Version B: page 8
Competence statements: y1, y2, y3, y4, y5, k11

E.g. a particle is projected at 20 m s^{-1} at an angle of 60^o to the horizontal from a cliff 50 m above sea level. Find

(i) the greatest height,
(ii) how far out it hits the sea,
(iii) the angle and speed at which it hits the sea.

This answer demonstrates the use of horizontal and vertical motion.



(i) Vertically: $u_y = 20\sin 60 = 10\sqrt{3} \approx 17.3, \ a = -g$

$v_y = 17.3 - 9.8t$:

at the top, $v_y = 0, \ 17.3 = 9.8t \Rightarrow t \approx 1.77\text{secs}$

$y = 17.3t - 4.9t^2 \Rightarrow s_y = 17.3 \times 1.77 - 4.9 \times 1.77^2 \approx 15.3$

$\Rightarrow$ Greatest height = 50 + 15.3 ≈ 65.3 metres

(ii) Vertically: $y = 17.3t - 4.9t^2$ and y = -50 at sea level.

$\Rightarrow -50 = 17.3t - 4.9t^2 \Rightarrow 4.9t^2 - 17.3t - 50 = 0$

$\Rightarrow t \approx \dfrac{17.3 \pm \sqrt{1280}}{9.8} \approx 5.42$ sec (taking the +ve root.)

Horizontally: $x = 20\cos 60t = 10 \times 5.42 \approx 54.2$ metres

(iii) Vertically: $v_y = u_y - 9.8t = 17.3 - 9.8 \times 5.42 \approx -35.8$

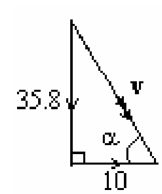
Horizontally: $v_x = u_x = 10$

$\Rightarrow$ Speed = $\sqrt{10^2 + 35.8^2}$

$= 37.1\,\text{ms}^{-1}$

$\Rightarrow$ Angle at sea = $\tan^{-1}\left(\dfrac{35.8}{10}\right)$

$\approx 74.4^0$



E.g. using vectors to solve the question above:

(i) $\mathbf{u} = \begin{pmatrix} 10 \\ 17.3 \end{pmatrix}, \ \mathbf{a} = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix}$

$\mathbf{v} = \mathbf{u} + \mathbf{a}t \Rightarrow \mathbf{v} = \begin{pmatrix} 10 \\ 17.3 \end{pmatrix} + \begin{pmatrix} 0 \\ -9.8 \end{pmatrix}t = \begin{pmatrix} 10 \\ 17.3 - 9.8t \end{pmatrix}$

$v_y = 0$ when $17.3 = 9.8t \Rightarrow t = \dfrac{17.3}{9.8} \approx 1.77$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2 = \begin{pmatrix} 10t \\ 17.3t \end{pmatrix} + \begin{pmatrix} 0 \\ -4.9t^2 \end{pmatrix} = \begin{pmatrix} 10t \\ 17.3t - 4.9t^2 \end{pmatrix}$

When $t = 1.77$ $\mathbf{s} = \begin{pmatrix} 17.7 \\ 15.3 \end{pmatrix}$,

giving greatest height $= 50 + 15.3 = 65.3$ m.

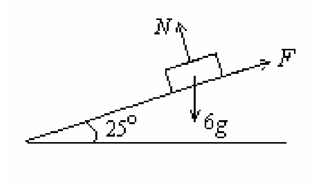
References:
Chapter 7
Pages 130-133

Exercise 7A
Q. 5

Resolving forces

Force is a vector. The technique of resolving vectors, and therefore forces, has been covered in chapter 5.
The component of R in a direction at an angle θ to the direction of R is $R\cos\theta$.
The component of R in a direction perpendicular to the line of action of R is 0, as cos90 = 0.

E.g. A body of mass 6 kg is being pulled at constant speed up a smooth slope of angle 25^0 to the horizontal by a force, F.



Resolving along slope
$F - 6g\sin 25 = 0$
$F = 24.8$ N
Resolving perpendicular to the slope:
$N - 6g\cos 25 = 0$
$N = 53.3$ N

Or, if **i**, **j** are along and perpendicular to the slope:

$$\begin{pmatrix} F - 6\sin 25 \\ N - 6g\cos 25 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Or, if **i**, **j** are horizontal and vertical:

$$\begin{pmatrix} F\cos 25 - N\sin 25 \\ F\sin 25 + N\cos 25 - 6g \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

References:
Chapter 7
Pages 134-135

Exercise 7B
Q. 3

Equilibrium

By Newton's 1st Law, if there is no acceleration in a given direction then there is no component of force in that direction.
A body is in equilibrium if there is no overall force in any direction. (i.e. the sum of resolved forces in any direction is zero.)
This applies to the particle model where all forces acting are considered to act through a point.

To check for equilibrium it is sufficient to check for no force in any two directions. Usually this will be in two perpendicular directions.

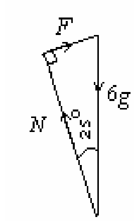
References:
Chapter 7
Pages 134-135

Triangle of forces

When there are three non-parallel forces acting on a body in equilibrium then the three forces may be represented in magnitude and direction by the sides of a triangle, taken in order. (See example.)

E.g. In the example above, the three forces are in equilibrium as there is no acceleration.

$$N = 6g\cos 25 = 53.3\,\text{N}$$
$$F = 6g\sin 25 = 24.8\,\text{N}$$



References:
Chapter 7
Pages 147-148

Exercise 7C
Q. 5

Newton's 2nd law in 2 dimensions

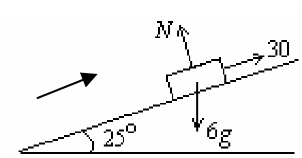
When a body is not in equilibrium then it will have an acceleration in a given direction.
This may be found by resolving or by using vectors
(**F** = m**a** is a vector equation)

i.e. $$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = m\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Or $F_1\mathbf{i} + F_2\mathbf{j} = m(a_1\mathbf{i} + a_2\mathbf{j})$

E.g. in the example above, the force, F, is increased to 30N. Find the acceleration and the value of N.

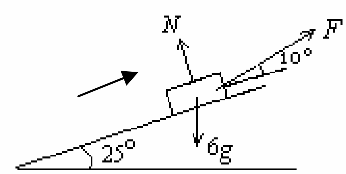
Resolving along slope:
$30 - 6g\sin 25 = 6a$
$\Rightarrow a = 0.86\,\text{m s}^{-2}$
Resolving perpendicular to the slope:
$N = 6g\cos 25 = 53.3\,\text{N}$



Forces in N

E.g. The force, F now makes an angle of 10^0 with the slope. How does this affect the value of N?

Resolving perpendicular to the slope:
$N + 30\sin 10 = 6g\cos 25$
$= 53.3\,\text{N}$
$\Rightarrow N = 53.3 - 5.2 = 48.1\,\text{N}$
N has decreased by 5.2N



Forces in N

Summary M1 Topic 8: **General motion**

References:
Chapter 8
Pages 157-164

Exercise 8A
Q. 3

Exercise 8B
Q. 3

Exercise 8C
Q. 2

Use of calculus

$$v = \frac{ds}{dt}; \qquad s = \int v \, dt$$

$$a = \frac{dv}{dt} = \frac{d^2 s}{dt^2}; \qquad v = \int a \, dt$$

Differentiate →

$s \quad v \quad a$

← *Integrate*

E.g. The velocity of a body is modelled by the formula $v = 3t^2 + 2t - 4$. It starts from the origin.
Find (i) the acceleration at time t,
(ii) the velocity at time t.

(i) $a = \frac{dv}{dt} = 6t + 2$

(ii) $s = \int v dt = \int (3t^2 + 2t - 4) dt$

$= t^3 + t^2 - 4t + c.$

Satisfied by $(0,0) \Rightarrow c = 0 \Rightarrow s = t^3 + t^2 - 4t$

Diagrams

Knowledge of C2 techniques of curve sketching (Straight lines, quadratics and higher polynomials) is important.

Displacement-time graph

- Velocity is the gradient of the curve
- Average velocity is the gradient of the chord between two times

Velocity-time graph

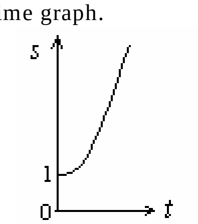
- Acceleration is the gradient of the curve
- Area under the curve is the displacement

Acceleration-time graph

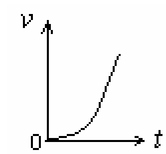
- Area under the curve is the velocity change

E.g. The displacement of a body is modelled by the formula $s = t^3 + 1$.
Sketch (i) the displacement - time graph,
(ii) the velocity-time graph and
(iii) the acceleration-time graph.

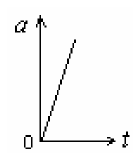
(i) $s = t^3 + 1$.
When $t = 0$, $s = 1$
When $s = 0$, $t = -1$



(ii) $v = 3t^2$
v increases with time.
When $v=0, t=0$



(iii) $a = 6t$
Acceleration increases at a constant rate, $a=0$ when $t=0$.



References:
Chapter 8
Pages 169-174

Exercise 8D
Q. 4

Using vectors

Formulae for uniform acceleration and calculus may be used with the variables modelling vectors in 2 or 3 dimensions.

e.g. If $\mathbf{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\mathbf{u} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ then when $t = 3$

Using $\mathbf{v} = \mathbf{u} + \mathbf{a}t \Rightarrow \quad \mathbf{v} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix} 3 = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$

When differentiating or integrating deal with the components.

e.g. If $\mathbf{v} = \begin{pmatrix} 3t^2 \\ 2t \end{pmatrix}$ then $\mathbf{a} = \frac{d\mathbf{v}}{dt} = \begin{pmatrix} 6t \\ 2 \end{pmatrix}$

and $\mathbf{s} = \int \begin{pmatrix} 3t^2 \\ 2t \end{pmatrix} dt = \begin{pmatrix} t^3 + c_1 \\ t^2 + c_2 \end{pmatrix}$

(where the constants of integration, c_1 and c_2, are found from initial conditions.)

E.g. The velocity of a body is modelled by the formula $v = (3t^2\mathbf{i} + 3\mathbf{j})$ ms^{-2}. Initially it is at the point with position vector $(\mathbf{i} - \mathbf{j})$ m.
Find (i) the acceleration at time t,
(ii) the displacement at time t,
(iii) the time when the particle is travelling in a direction parallel to the x-axis.

(i) $\mathbf{a} = \frac{d\mathbf{v}}{dt} = 6t\mathbf{i}$ ms^{-2}

(ii) $\mathbf{s} = \int \mathbf{v} dt = \int (3t^2\mathbf{i} + 3\mathbf{j}) dt = t^3\mathbf{i} + 3t\mathbf{j} + \mathbf{c}$

When $t = 0$, $\mathbf{s} = (\mathbf{i} - \mathbf{j}) \Rightarrow \mathbf{c} = (\mathbf{i} - \mathbf{j})$

$\Rightarrow \mathbf{s} = (t^3 + 1)\mathbf{i} + (3t - 1)\mathbf{j}$

(iii) When $\mathbf{s} = k\mathbf{i}$, $3t - 1 = 0 \Rightarrow t = \frac{1}{3}$.