Mechanics 2 : Revision Notes

1. Kinematics and variable acceleration

Displacement (x) → Velocity (v) → Acceleration (a)

x = f(t) —— differentiate ——→ v = $\frac{dx}{dt}$ —— differentiate ——→ a = $\frac{dv}{dt} = \frac{d^2x}{dt^2}$

Acceleration → Velocity (v) → Displacement x

a —— integrate ——→ v = $\int a\, dt$ —— integrate ——→ x = $\int v\, dt$

DON'T' forget +c when integrating

If TRIG functions are involved you need to work in radians

You may be asked to use the ACCELERATION to calculate the
Force acing on a particle at a given time using **Force = mass x acceleration**

In two or 3 dimensions

– differentiate each of the i,j k components as individual functions but don't forget to include the i,j,k in your answer with the new functions

- Integrating – you need '+c' in each of the I,j,k components which you can then evaluate using given conditions

Example

*A particle moves so that at time t seconds it's velocity **v** ms^{-1} is given by* **v** = $(4t^3 - 12t)\boldsymbol{i} + 5\boldsymbol{j} + 8t\boldsymbol{k}$

*When t = 0, the position vector of a particle is (-2**i** + 4**k**)*

$$x = (t^4 - 6t + c_x)\boldsymbol{i} + (5t + c_y)\boldsymbol{j} + (4t^2 + c_z)\boldsymbol{k}$$

t = 0

x = -2i + 4k $c_x = -2$ $c_y = 0$ $c_z = 4$

$$x = (t^4 - 6t - 2)\boldsymbol{i} + 5t\boldsymbol{j} + (4t^2 + 4)\boldsymbol{k}$$

Find the time when the particle is moving…….. you need to consider velocity and its components

- Moving due west – parallel to the i direction **j component of velocity = 0**

- Moving due north – parallel to the j direction **i component of velocity = 0**

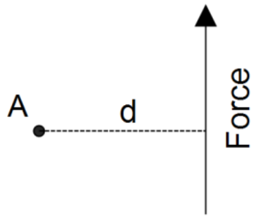
MAGNITUDE – if you are asked to find the **magnitude** of the velocity (SPEED) or acceleration or force at a given time - you need to use **Pythagoras**

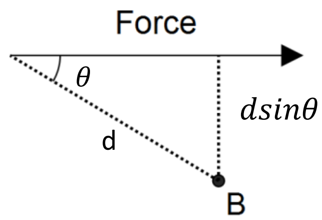
2. Moments and Equilibrium

(force x perpendicular distance from the point to the force)

Moment of a force about A = Fd

Moment of the force about B = $Fd\sin\theta$





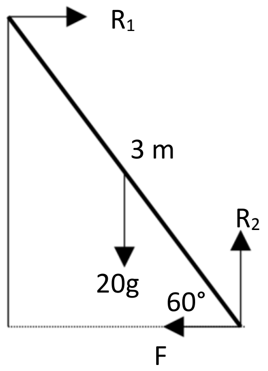
SI units of a moment are *Nm*
Clockwise – negative
Anticlockwise – positive

> Equilibrium : For a rigid body to be in equilibrium
> - the **resultant** force must be zero
> - the **total moment** of all the forces must be zero

In solving problems
- draw a diagram to show all the forces – weight – 'normal' – tensions ….
- by taking moments about a point you can ignore the forces acting at that point
- resolve – vertically and horizontally

Example
A ladder of length 3 m and mass 20 kg, leans against a smooth, vertical wall, so that the angle between the horizontal ground and the ladder is 60. Find the magnitude of the friction and the normal reaction forces that act on the ladder, if it is in equilibrium.

Step 1: **Draw a diagram showing all of the forces**



Step 2 : **Resolving Vertically**

$R_2 = 20g \quad R_2 = 196$ N

Step 3 : **Taking moments at the base**

$20g \times 1.5\cos 60° = R_1 \times 3\sin 60°$

$R_1 = 56.6$ N

Step 4 : **Resolving horizontally**

$F = R_1 = 56.6$ N

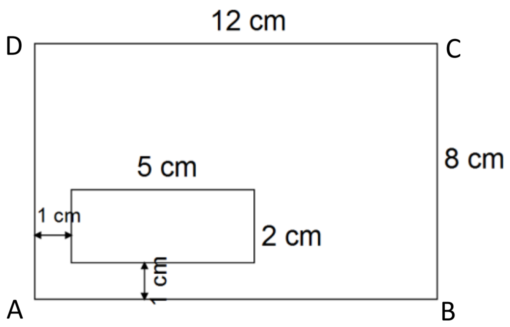
3. Centres of Mass

- Use a table to set out your workings
- Make sure you show the values substituted into the formula $\bar{x} = \frac{\sum mx}{\sum m}$ or equivalent
- '**Light'** means you can ignore the 'weight' of the framework connecting the particles
- When a body is **suspended in equilibrium** from a point the centre of mass is directly below the point of suspension

Centre of mass of a lamina

- When working with a uniform lamina you need to work with the area instead of mass (as the area of each part will be proportional to its mass)

Example
The diagram shows a uniform rectangular lamina that has had a hole cut in it. The centre of mass of the lamina is a distance x from AD and a distance y from AB. Find x and y.



	Large Rectangle	**Small Rectangle**	**Lamina**
Area	96 cm^2	10cm^2	86cm^2
x (from AD)	6 cm	3.5 cm	x
y (from AB)	4 cm	2 cm	y

Large rectangle = small rectangle + lamina

$96 \times 6 = 86x + 10 \times 3.5$
x = 6.30 cm

$96 \times 4 = 86y + 10 \times 2$
y = 4.23 cm

4. Energy ,Work and Power

Kinetic energy = $\frac{1}{2}mv^2$	**Every moving body has kinetic energy**
Work done by a force = Fs	(either increase or decrease the total energy of the system)

Example
A box is initially at rest on a smooth horizontal surface. The mass of the box is 10kg. A horizontal force of magnitude 15 N acts the box as it slides 6 m. Find the speed of the box when it has travelled 6 m.

Initial : KE = 0 (box is at rest)

Change in energy due to work done = 15 × 6 = J
After 6m KE = 90 J

$\frac{1}{2}10v^2 = 90 \qquad v = 4.24ms^{-1}$

Potential energy = *mgh* **work done against gravity - as a particle is raised increases PE**

Example
A stone of mass 0.5kg is thrown over a cliff at a speed of 4 ms^{-1}. It hits the water at a speed of 15ms^{-1}. Assuming that there is no resistance to the motion of the stone calculate the height of the cliff.

Initially:
KE = $\frac{1}{2} \times 0.5 \times 4^2$
= 4 J

When it hits the water
KE = $\frac{1}{2} \times 0.5 \times 15^2$
= 56.25 J

Gain in KE = loss in PE
0.5×g×h=56.25
h = 11.5 m

Elasticity – Hooke's Law

$T = \frac{\lambda e}{l}$ e – extension *l* = natural length λ – modulus of elasticity

$EPE = \frac{\lambda e^2}{2l}$ elastic potential energy of a stretched or compressed spring

Derived using work done by a variable force $\int f(x)dx$ $so\ EPE$= $\int_0^e \frac{\lambda e}{l}$ de

- A particle will not remain at rest if tension (due to elasticity) is present
- Maximum extension will occur when velocity = 0 (turning point)
- At maximum speed – acceleration = 0 (if vertical problem T = mg)

Example
A sphere of mass 150 grams is attached to one end of an elastic string of natural length 50 cm and modulus of elasticity 5 N. The sphere is released from rest at O and falls vertically. Determine the maximum speed of the sphere.

At max speed – acceleration = 0 resultant force = 0 so T = mg

$\frac{5e}{0.5} = 0.15 \times 9.8$ e = 0.147 m

At this point GPE lost = KE gained + EPE gained

GPE lost : Change in height= 0.5 + 0.147 m
Loss in GPE = 0.15×9.8×0.647
= 0.951 J

Gain EPE = $\frac{5(0.147)^2}{2\times.05} = 0.108$ J so gain in KE = 0.951 – 0.108 = 0.843 J

$\frac{1}{2} \times 0.150 \times v^2$ = 0.843

v = 3.35 ms^{-1}

POWER (measured in Watts)

Power = $\frac{work\ done}{time\ taken}$ rate of doing work/rate of gain of kinetic energy

Power = Fv

5. Motion in a circle

Angular speed $\boldsymbol{\omega}$ – measures the angle (radians) turned though per second (rad s^{-1})

Remember in a full turn there are 2π radians

Velocity has magnitude $r\omega$ (r = radius) and is directed along a tangent to the circle

Acceleration = $\boldsymbol{r}\omega^2$ or $\frac{v^2}{r}$ and is directed towards **the centre of the circle**

Movement in a horizontal circle with constant speed

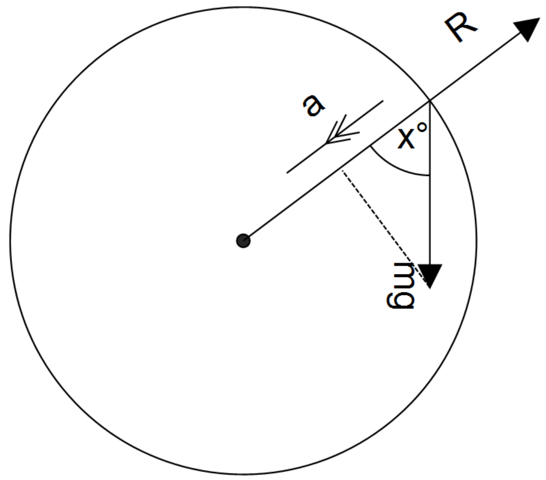
- Resultant of vertical forces = 0
- F = ma can be applied radially – acceleration acting towards the centre

Movement in a vertical circle

- Use of KE = ½ mv^2 and GPE= mgh to calculate v (use this to calculate a)
- MAXIMUM speed – at lowest point
- FULL REVOLUTION v at the highest point ≥ 0
- A particle travelling on a circle will leave the circle when R = 0

Remember to split the 'weight' of a particle so that you are resolving parallel to the acceleration when using F = ma

mg cos x – R = ma



6. Differential Equations

Forming Equations

- Use F = ma to find the acceleration
- Use $a = \frac{dv}{dt}$ to form the differential equation
- If necessary rearrange into the form given – shows your steps clearly

Solving

- Separate the variables
- Integrate both 'sides' REMEMBER to include the + c
- Use given conditions to calculate c (*initially implies t = 0*)

Useful integrals

$$\int \frac{1}{v} \; dv = \ln|v| + c \qquad \int \frac{1}{av+b} \; dv = \frac{1}{a}\ln|av+b| + c$$

$$\int \sin av \; dv = -\frac{1}{a}\cos av + c$$

$$\int \cos av \; dv = \frac{1}{a}\sin av + c$$