MEI

Mathematics in Education and Industry

MEI Structured Mathematics

Module Summary Sheets

Mechanics 2

(Version B: reference to new book)

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References:
Chapter 1
Pages 1-3

Example 1.1
Pages 4-5

Exercise 1A
Q. 4

Example 1.2
Page 5

Exercise 1A
Q. 2

Coulomb's Laws

1. Friction opposes relative motion between two surfaces in contact.
2. Friction is independent of the relative speed of the surfaces.
3. The magnitude of the frictional force has a maximum value which depends on the normal reaction between the surfaces and the roughness of the surfaces in contact.
4. If there is no sliding between the surfaces $F \le \mu R$ where F is the frictional force and R is the normal reaction. μ is called the coefficient of friction.
5. When sliding is about to occur, friction is said to be limiting and $F = \mu R$.
6. When sliding occurs $F = \mu R$.
7. For any pair of surfaces, μ is constant.

References:
Chapter 1
Pages 4-8

Example 1.3
Page 6

Exercise 1A
Q. 8, 9

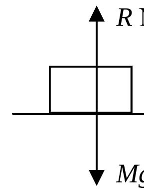
Modelling with friction

It is often satisfactory to model a situation as having negligible friction. We then describe the surfaces as 'smooth'. Otherwise the surfaces are described as 'rough'.

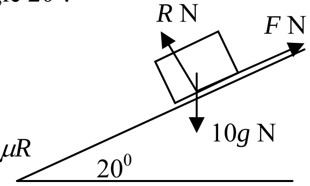
Note that 'smooth' surface and a 'smooth' curve are not the same thing.

E.g. a block is placed on a rough surface where $\mu = 0.8$.

(1) The surface is flat.
There are no horizontal forces acting and so there will be no friction.



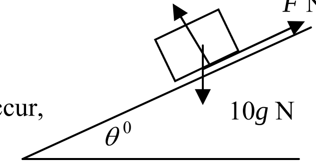
(2) The surface is a slope of angle 20^0.



Resolving perpendicular to the slope:
$R = 10g\cos 20 = 92.1$
The maximum value of F is μR
$F = 92.1 \times 0.8 = 73.7$ (3 s.f.).

The component of weight down the slope is $10g\sin 20 = 33.5$
Since $33.5 < 73.7$, there will be no motion and so
$F = 33.5$ (3 s. f.).

(3) The slope is θ^0 such that sliding is about to occur. Find θ^0.

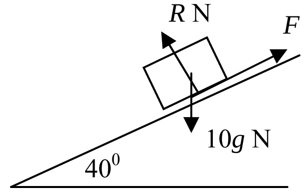


As before: $R = 10g\cos\theta$
No motion along slope means that
$10g\sin\theta = F$

As sliding is about to occur,
$F = \mu R$

$\Rightarrow 10g\sin\theta = \mu 10g\cos\theta$
$\Rightarrow \tan\theta = 0.8$
$\Rightarrow \theta = 38.7^0$.(3 s. f.).

(4) The slope is 40^0. Find the acceleration.



As before: $R = 10g\cos 40$

Maximum $F = \mu R$
$= 0.8 \times 10g\cos 40 = 8g\cos 40$.

So applying Newton's 2nd law

$10g\sin 40 - 8g\cos 40 = 10a$
$\Rightarrow 10a = 63.0 - 60.1 = 2.9$
$\Rightarrow$ acceleration is 0.29 m s^{-2} (2 s.f.)

Note: If a turns out to be negative at this stage you know there is no sliding, $a = 0$ and $F \ne \mu R$;
$F = 10g\sin\theta$

Summary M2 Topic 1: ***(B) Moments of Forces***

References: Chapter 3 Pages 25-27

Rigid bodies

The models used so far assume that a body is a particle (i.e. has no dimensions) so that all forces acting on it pass through a single point. This is not always satisfactory and a development of the model is to assume that the body has size and is rigid (is not deformed when acted on by a force). The weight of the body is taken to act through a point called the centre of mass.

References: Chapter 3 Pages 27-29

Exercise 3A Q. 2

Moments

If a force does not act through a point then it has a turning effect - or **moment** - about that point.
The moment of a force about a point is defined as :

$$\text{Moment} = Fd$$

where d is the perpendicular distance of the point from the line of action of the force, F. In two dimensions, moments can be clockwise or anti-clockwise.

References: Chapter 3 Page 29

Couples

If there is no overall resultant force, then the body will have no linear acceleration. Two forces equal in magnitude, opposite in direction and acting along parallel but different lines will have a turning effect called a couple.

References: Chapter 3 Pages 29-32

Exercise 3A Q. 7

Equilibrium

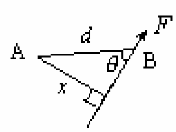
When all the forces on a body act through a point, the body is in equilibrium if there is zero resultant force.
When the forces do not all act through the same point, the body is in equilibrium only if the total moment about every point is also zero.
When the resultant force is zero, if the moment of all the forces about any one point is zero then it will also be zero about every other point.

References: Chapter 3 Page 40-45

Exercise 3B Q. 1, 7

Moment of a forces at angles

When d is the distance from a point A to some point B on the line of action of the force F such that AB makes an angle of θ with F, the moment of F about A is $Fx = Fd\sin\theta$.



Note that this is equivalent to considering the moments about A of the resultant parts of F in directions parallel and perpendicular to AB.

References: Chapter 3 Pages 53-55

Exercise 3C Q. 2, 5

Sliding and toppling

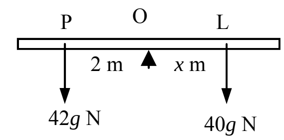
Two surfaces slide when the limiting frictional force is insufficient to maintain equilibrium.

(*) The resultant normal reaction, N, between two surfaces must pass through a point of contact of the surfaces.

If the set-up is slowly changed and limiting friction is reached while the condition (*) holds, sliding takes place first. If not the body topples first.

E.g. Paul has mass 42 kg , Lewis 40 kg and Clare 30 kg. They are playing on a see-saw.

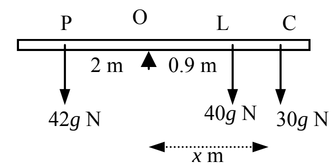
1. Paul sits 2 m from the fulcrum. Where does Lewis need to sit in order to balance the see-saw?



M(O) a.c.: $42g \times 2 - 40gx = 0$

$\Rightarrow x = 2.1$ Lewis sits 2.1 m from O.

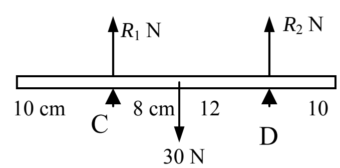
2. Paul sits 2 m from the fulcrum. Lewis sits on the other side, 0.9 m from the fulcrum. Where does Clare have to sit to balance the see-saw?



M(O) c.w.: $40g \times 0.9 + 30gx - 42g \times 2 = 0 \Rightarrow 30gx = 48g$

$\Rightarrow x = 1.6$ Clare sits 1.6 m from O.

E.g. A bar of length 40 cm and weight 30 N is supported horizontally by two supports, 10 cm from reach end. The centre of mass of the bar is 18 cm from the left-hand end.
Find the reaction forces on the supports.



R(↑): $R_1 + R_2 = 30$

M(D) c.w.: $20R_1 - 12 \times 30 = 0$

$\Rightarrow R_1 = 18$ and $R_2 = 12$

E.g. A uniform bar AC of mass 12 kg and length 80 cm is freely hinged on a wall. It is held horizontally by a wire fixed to the wall 60 cm above the hinge. Find the tension in the wire.

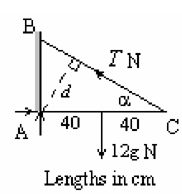
$M(A)$ c.w.: $40 \times 12g - Td = 0$

$d = 80 \times \sin\alpha = 48$

$\Rightarrow 48T = 480g$

$\Rightarrow T = 98$

The tension is 98 N.



References:
Chapter 7
Pages 143-147

Frameworks

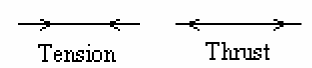
A framework is an arrangement of rods or struts hinged (or "pin-jointed") at the ends of the rods.

Example 7.2
Page 146

Simplifying assumptions

1. All parts of the framework are light.
2. All the pin-joints are smooth.
3. Any loads or other external forces are attached at the pin-joints.

The result of these assumptions is that all internal forces are directed along the rods. The forces are tension or thrust (compression).



Method of solution

1. Label all the internal forces. They may be marked as tensions or thrusts but you may find it better to mark them all as tensions; any negative values will be thrusts.

2. By resolving and taking moments, as far as possible calculate the external forces on whole system.

3. Take each pin-joint in isolation and find horizontal and vertical equilibrium equations for all the forces acting (including the internal forces) .

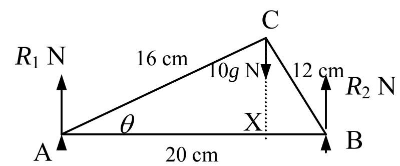
Plan the order of visiting the pin-joints to keep the number of unknown forces to a minimum at each stage.

4. When up-dating your diagram, remember keep the sign of each force relative to the direction marked.

5. In general there will be a redundant point i.e. to find all the forces it will be necessary to work at all but one of the pin-joints. The values already found may be checked for this point.

Exercise 7A
Q. 2, 8

E.g. The framework ABC as shown is in a vertical plane and is made up of three light rods freely pin-jointed at A, B and C. A mass of 10 kg is hung from C. Taking $g = 10$ m s^{-2}, find the normal reactions at A and B. Find also the forces in the rods and determine whether they are in tension or in thrust.



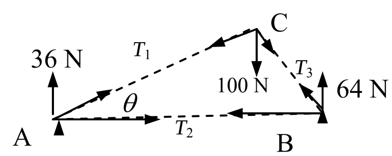
There will be upward forces at A and B, R_1 N and R_2 N respectively. The angle at C is 90^0 because 12:16:20=3:4:5

R($\uparrow$): $R_1 + R_2 = 100$

M(A) c.w.: $100.AX - 20R_2 = 0$

$AX \;=\; 16\cos\theta$ with $\cos\theta = \dfrac{16}{20} \Rightarrow AX = 12.8$

$\Rightarrow R_2 = \dfrac{1280}{20} = 64$ and $R_1 = 36$



Finding internal forces by looking at each point:

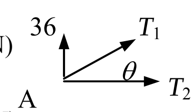
At A:

R($\uparrow$): $36 + T_1 \sin\theta = 0$

$\Rightarrow T_1 = -60$ (i.e. thrust of 60 N)

R($\rightarrow$): $T_2 + T_1 \cos\theta = 0$

$\Rightarrow T_2 = 48$ (i.e. tension of 48 N)

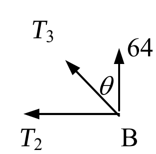


At B:

R($\uparrow$): $64 + T_3 \cos\theta = 0$

$\Rightarrow T_3 = -80$ (i.e. thrust of 80 N)

T_2 can be found by resolving horizontally as a check of the above.

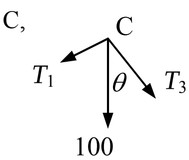


T_3 is found without looking at C, but this can be a useful check.

At C:

R($\uparrow$): $100 = 60\sin\theta + T_3 \cos\theta$

$\Rightarrow T_3 = 80$



(Alternatively:
R($\rightarrow$): $60\cos\theta = T_3 \sin\theta$
$\Rightarrow T_3 = 80$)

References:
Chapter 5
Pages 87-94

Exercise 5A
Q. 1(i), 2(ii), 4

Energy and Work

The energy of an object is changed when it is acted on by an unbalanced force.
When a force moves an object, the force is said to do work.

Work done = force × distance moved in direction of force

The SI unit of work is the joule, J (N m).

Kinetic energy is energy possessed because of motion. For the linear motion of a body of mass m and speed v.

$$\text{Kinetic Energy} = \frac{1}{2}mv^2$$

The work-energy principle
Work done by resultant force
= increase in kinetic energy of object

Examples

1. Work done by a force of 200 N in moving an object 3 m = 600 Joules.
2. A body increases speed due to a force of 30 N acting for 3 m.
   Kinetic Energy gained = 30x3 = 90 J
3. If the body above has mass 4 kg and was at rest then
   $$\text{Kinetic Energy} = \frac{1}{2}\times 4\times v^2 - 0 = 90$$
   $\Rightarrow v = \sqrt{45}$. Final speed is 6.71 m s^{-1} (3 s. f.)
4. A car of 1.5 tonnes is brought to rest in 100 m from a speed of 30 m s^{-1} by the brakes exerting a constant force. Find the force.
   Work done = Final K.E. – Initial K.E.
   $$\Rightarrow 100F = \frac{1}{2}\times 1500\times 30^2 - 0$$
   $$\Rightarrow F = \frac{15\times 900}{2} = 6750.$$ Force is 6750 N

References:
Chapter 5
Pages 96-100

Potential Energy

Potential energy is energy stored because of position. In this unit it is the work done against gravity when an object is raised and the work done by gravity when it is lowered.

P.E. = mgh

References:
Chapter 5
Page 97

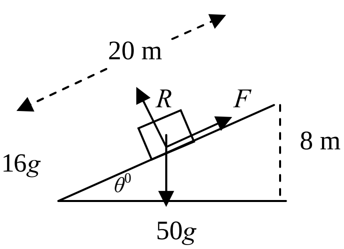
Conservation of mechanical energy

When gravity is the only force which does work, mechanical energy is conserved and
Kinetic energy + Potential energy = constant

Examples

1. A sledge carrying a child, with a total mass of 50 kg, slides from rest down a smooth snow slope of length 20 m. The slope drops 8 m vertically. Find the speed at the bottom of the slope.

$F = 0$, $u = 0$ and $h = 8$.
$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mgh$$
$$\Rightarrow \frac{1}{2}mv^2 = 8mg \Rightarrow v^2 = 16g$$
$$\Rightarrow v \approx 12.5 \text{ m s}^{-1}.$$



2. The snow slope is not smooth and μ = 0.3. What now will be the speed at the bottom of the slope?

$F = \mu R$ and $R = mg\cos\theta$
$$\Rightarrow F = \mu mg\cos\theta \text{ where } \sin\theta = \frac{8}{20}.$$
$$\Rightarrow (mg\sin\theta - \mu mg\cos\theta)\times 20 = \frac{1}{2}mv^2 - 0$$
$$\Rightarrow 8g - 6g\cos 23.58 = \frac{1}{2}v^2$$
$$\Rightarrow v^2 \approx 49 \Rightarrow v \approx 7.$$ Speed ≈ 7 m s^{-1}.

References:
Chapter 5
Pages 101-102

Exercise 5B
Q. 8

Work and kinetic energy for 2-dimensional motion

A force acting at right angles to the direction of motion does no work.
If a force is acting at an angle to the direction of motion then resolve into two forces, along the direction of motion and perpendicular to it.
NB The work done is calculated as
either force × dist moved in direction of force
or dist moved × component of force in direction of displacement.

References:
Chapter 5
Pages 109-111

Exercise 5C
Q. 3, 9

Power

Power is the rate at which work is done.

$$P = \frac{Fs}{t} = Fv$$

for a constant force, F.

E.g. A mass of 10 kg is being raised at 2 m s^{-1}.
What is the power required?

$F = 10g$: power = $10g\times 2 = 20g$ = 196 W

Impulse and momentum – 1

References: Chapter 6 Pages 118-123

Exercise 6A Q. 1(i), 2(i)

Exercise 6A Q. 8, 10

Impulse and Momentum

The ***impulse*** given by a constant force is defined as

Impulse = Force × time

The ***linear momentum*** of a body is defined as $m\mathbf{v}$. **(**In this unit linear momentum is simply called momentum.)
A force acting on a body changes its velocity and hence its momentum.

If a force acts for time t and changes the velocity from $\mathbf{u}$ to $\mathbf{v}$**,** then

Impulse = final momentum – initial momentum

$$\mathbf{J} = \mathbf{F}t = m\mathbf{v} - m\mathbf{u}.$$

Impulse and momentum are both vector quantities.

The impulse of a variable force is given by

$$\int_0^T \mathbf{F}\,dt = m\mathbf{v} - m\mathbf{u}$$

The units of momentum and impulse are the N s.

Magnitude of momentum of

(a) a car of 1.2 tonnes moving at 3 m s^{-1} is $1200 \times 3 = 3600$ N s,

(b) a of hockey ball of 0.18 kg moving at 20 m s^{-1} is $0.18 \times 20 = 3.6$ N s.

Magnitude of impulse of force of 500 N acting for 2 sec is 1000 N s.

A hockey ball of mass 0.18 kg is slowed down by resistance to the ground from 20 m s^{-1} to 10 m s^{-1} in 4 seconds.

Take J, the impulse and F, the friction, to be in the direction of motion:

$J = 0.18\times10 - 0.18\times20 = -1.8$

Now $4F = -1.8 \Rightarrow F = -0.45$ N.

Resistance opposes motion with a force of 0.45 N.

E.g. The direction of a hockey ball of 0.18 kg is changed by a force acting for 1.2 sec. The original velocity was $(12\mathbf{i} + 13\mathbf{j})$ m s^{-1} and the final velocity is $(24\mathbf{i} + 5\mathbf{j})$ m s^{-1}. Find the magnitude, F N, of the force.

$1.2\,\mathbf{F} = 0.18\big((24\mathbf{i}+5\mathbf{j})-(12\mathbf{i}+13\mathbf{j})\big) = 0.18(12\mathbf{i}-8\mathbf{j})$

$\Rightarrow \mathbf{F} = 1.8\mathbf{i} - 1.2\mathbf{j}$

$\Rightarrow |\mathbf{F}| = \sqrt{1.8^2+1.2^2} = 2.16$ N (3 s. f.)

References: Chapter 6 Pages 128-134

Exercise 6B Q. 2

Conservation of momentum

When there are no external forces, the total momentum of the system is conserved.

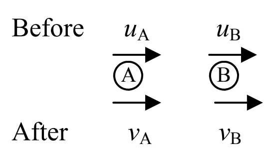
$$m_A\mathbf{v}_A + m_B\mathbf{v}_B = m_A\mathbf{u}_A + m_B\mathbf{u}_B$$

References: Chapter 6 Pages 138-142

Exercise 6C Q. 5

Coefficient of restitution

Newton's Experimental Law (Law of Impact) for direct impact:



$$\frac{\text{speed of separation}}{\text{speed of approach}} = \text{constant.}$$

We write $\dfrac{v_B - v_A}{u_B - u_A} = -e.$

The constant, e, is called the coefficient of restitution.

For all pairs of surfaces, $0 \le e \le 1$

1. A railway truck of mass 25 tonnes is moving along a horizontal track at 7 m s^{-1} when it collides with a stationary truck of mass 10 tonnes. They join and move on along the track together. With what speed do they move?
Momentum is conserved so:

$35v = 25\times7 + 10\times0 \Rightarrow v = 5$

$\Rightarrow$ speed is 5 m s^{-1}.

(Notice that the units of mass are unimportant so long as they are consistent.)

2. If it is the lighter truck that is moving at 7 m s^{-1} and it collides with the heavier truck which is stationary, then with what speed do they move?

$35v = 10\times7 + 25\times0 \Rightarrow v = 2$

$\Rightarrow$ speed is 2 m s^{-1}.

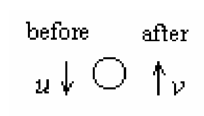
References: Chapter 6 Page 139

Exercise 6C Q. 3

Direct impact with a fixed surface

The speed of the fixed surface before and after the impact is 0.

i.e. $v = eu$.



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References:
Chapter 6
Page 131

Example 6.10
Page 131

Exercise 6C
Q. 9

Direct collision in a straight line

See diagram on previous sheet.
By Newton's experimental law

$$\frac{v_B - v_A}{u_B - u_A} = -e.$$

If A catches up with B then $u_A > u_B$ but after the collision $v_B > v_A$.

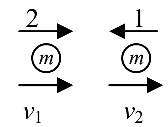
By conservation of momentum

$$m_A v_A + m_B v_B = m_A u_A + m_B u_B$$

Given sufficient information, the two equations may be solved simultaneously.

E. g. Two balls of mass m moving in opposite directions at 2 m s^{-1} and 1m s^{-1} hit each other directly. The coefficient of restitution is 0.5.

Before



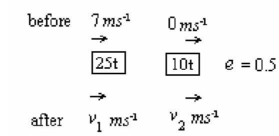
e = 0.5

After

N.E.L: $v_2 - v_1 = -0.5(-1 - 2) = 1.5$
Cons. of momentum: $mv_2 + mv_1 = 2m + (-1)m$
$\Rightarrow \quad v_2 + v_1 = 1$
Solving simultaneously: $v_1 = -0.25$, $v_2 = 1.25$
The directions of motion are reversed and the second ball now moves faster.

If, in the case of the two trucks (see previous page), the coefficient of restitution between the two trucks is 0.5 then find the velocities in the two cases above after the collision.

1.



(as before) $25v_1 + 10v_2 = 25 \times 7 + 10 \times 0$ (Cons. of mom.)
$v_2 - v_1 = -0.5(0-7) = 3.5$ (NEL))
so $25v_1 + 10v_2 = 175$
and $v_2 - v_1 = 3.5$
$\Rightarrow v_2 = 7.5, v_1 = 4$

2. Before 0 m s^{-1} → 7 m s^{-1} ←

$25v_1 + 10v_2 = 25 \times 0 - 10 \times 7$
$v_2 - v_1 = -0.5(-7 - 0) = 3.5$
$\Rightarrow v_1 = -3$, $v_2 = 0.5$ (The lighter truck rebounds.)

In the second case:
Impulse on 10000 kg truck
= 10 000 (0.5–(–7)) Ns
= 75000 Ns towards the right.

K.E. lost on impact

$$= \frac{1}{2} \times 25000(0^2 - 3^2) + \frac{1}{2} \times 10000 \times (7^2 - 0.5^2)$$

$= 131250$ J ≈ 1.31 KJ (3 s. f.).

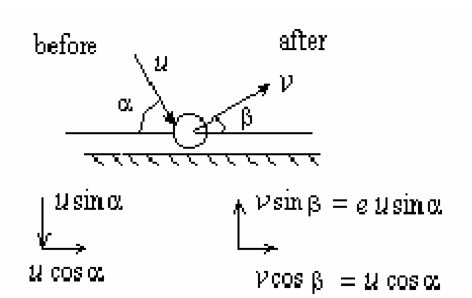
References:
Chapter 6
Pages 137-139

Exercise 6D
Q. 5, 10

Oblique Impact with a smooth plane

When an object hits a smooth plane there can be no impulse parallel to the plane so velocity is unchanged in this direction.
Newton's experimental law (NEL) applies to the components of velocity in the direction perpendicular to the plane.

When a ball is travelling with speed u at an angle of α to the plane:
Original velocity perp. to the plane = $u\sin\alpha$
Final velocity perp. to the plane = $ue\sin\alpha$
Original velocity parallel to the plane = $u\cos\alpha$
Final velocity parallel to the plane = $u\cos\alpha$



If angle of incidence = α , angle of reflection = β

$$\tan\beta = \frac{ue\sin\alpha}{u\sin\alpha} = e\tan\alpha \Rightarrow e = \frac{\tan\beta}{\tan\alpha}$$

Impulse = Final momentum - initial momentum
Since the velocity parallel to the plane is unchanged the momentum in this direction is unchanged and so Impulse = 0.

Perpendicular to the plane the impulse is
$meu\sin\alpha - (-u\sin\alpha) = (1+e)mu\sin\alpha$

E.g. a ball is moving at 3 m s^{-1} and bounces off a smooth wall. The angle of incidence is 40^0 and the coefficient of restitution is 0.7.
Find the speed after the bounce and the angle of reflection.

$\tan\beta = e\tan\alpha \Rightarrow \tan\beta = 0.7 \times \tan 40 = 0.5874$
$\Rightarrow \beta = 30.4^0$.
$v = \sqrt{(3\cos 40)^2 + (3 \times 0.7 \times \sin 40)^2} = 2.67$
The angle of reflection is 30.4^0
and the new speed is 2.67 m s^{-1} (3 s.f.).

References:
Chapter 4
Pages 63-67

Centre of mass (c. m.)

This is the point through which the weight of the whole body may be considered to act.
For the particle model this is the point through which all the forces on the body act.
For the rigid body model, the centre of mass is the 'balance point'.

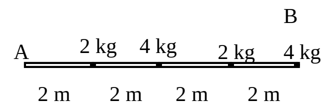
Example 4.3
Page 66

For particles on the *x*-axis, the position, $\bar{x}$, of the centre of mass relative to a point O is such that the moment of the total mass about the point O is equal to the sum of the moments of the separate masses about O.

i.e. $\left(\sum_{\text{all } i} m_i\right)\bar{x} = \sum_{\text{all } i} m_i x_i$

Exercise 4A
Q. 2, 4

E.g. A light rod has masses attached as shown. Find the centre of mass.



Total mass = 12
If centre of mass is at $\bar{x}$ from A, then
$12\bar{x} = 2\times 2 + 4\times 4 + 2\times 6 + 4\times 8$
$\Rightarrow 12\bar{x} = 64$
$\Rightarrow \bar{x} = \frac{16}{3}$

Uniform bodies

A uniform body is one made from identical material throughout. It follows that the centre of mass will be at a point of symmetry, if there is one.
e.g. a rectangular sheet with constant thickness and density will have its centre of mass where its diagonals intersect.

Exercise 4A
Q. 6, 8

Laminas
A lamina is a layer (or sheet) whose thickness is negligible.

Centre of mass for some shapes

Solid cone or pyramid	$\frac{1}{4}h$ from base.
Hollow cone or pyramid	$\frac{1}{3}h$ from base.
Solid hemisphere	$\frac{3}{8}r$ from base.
Hollow hemisphere	$\frac{1}{2}r$ from base.
Semi-circular lamina	$\frac{4r}{3\pi}$ from base.

See the handbook.

Centre of mass for two and three-dimensional bodies

References:
Chapter 4
Pages 70-72

Suppose a body consists of point masses m_i at points (x_i, y_i)

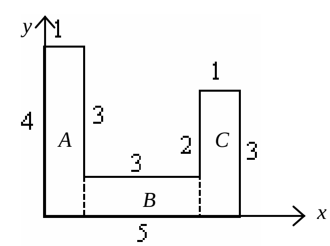
The centre of mass will be at $(\bar{x}, \bar{y})$
where $M\bar{x} = \sum m_i x_i$, $M\bar{y} = \sum m_i y_i$ and $M = \sum m_i$.

Similarly for 3 dimensions.

Exercise 4B
Q. 2, 5

The centre of mass may also be thought of as the mean position vector of the particles 'weighted' by the masses.

E.g. Find the centre of mass, referred to the axes in the diagram, of the given uniform lamina, *L*.



	A	*B*	*C*	*L*
m	4	3	3	10
x	0.5	2.5	4.5	$\bar{x}$
y	2	0.5	1.5	$\bar{y}$

$10\bar{x} = 4\times 0.5 + 3\times 2.5 + 3\times 4.5 \Rightarrow \bar{x} = 2.3$
$10\bar{y} = 4\times 2 + 3\times 0.5 + 3\times 1.5 \Rightarrow \bar{y} = 1.4$

Composite bodies

1. If there is a line of symmetry then the centre of mass will lie on that line.
2. If the body consists of separate parts then the c. m. of the whole body can be calculated from point masses with the masses of the parts at the centres of mass of the parts.

$$M_{B+C}\,\bar{\mathbf{r}} = M_B\,\bar{\mathbf{r}}_B + M_C\,\bar{\mathbf{r}}_C$$

Using Experimental Results

This topic relates to the former coursework component of this unit and has been retained because of its general importance and interest. There are no competence statements as such, but elements of the ideas of modelling and testing against experimental results will pervade the examination.

References:
Chapter 2
Pages 14-16

Using an experiment to test a model

A model is a mathematical description of a situation in the real world in which assumptions are made to enable the problem to be solved mathematically.
When a solution is found it has to be checked against reality.
If the match is not good then it may be possible to amend (or refine) the model.
This process uses the MEI modelling flowchart. (Students should ask their teacher for this.)

References:
Chapter 2
Pages 16-17

Variation in measurements

When a model is tested with an experiment, measurements will be taken. Because of error in measurement and/or variability in the situation there will be some variation in these measurements. This variation may be random or systematic.
Consequently, if the experiment is repeatable, a number of measurements should be taken and the mean value found. The maximum and minimum values should be used to estimate the error bounds.

References:
Chapter 2
Pages 18-19

Comparing with the model

The solution for the model will give a set of predicted results. These values have to be compared with experimental values, and this is often done most conveniently using graphs.

References:
Chapter 2
Pages 20-21

Example 2.1
Page 20

Exercise 2A
Q. 1

Errors related to significant figures and decimal places

Care has to be taken with the precision of measurements and also the values assumed to be constant. e.g. masses assumed to be as stated by the makers or taking $g = 9.8 \text{ m s}^{-2}$.

It may be that the predictions from the model would remain consistent with increased precision in the measurements made.

References:
Chapter 2
Page 21

Exercise 2A
Q. 5, 9

Percentage error

If x is the true value of a measurement and X is the approximated (measured) value then

error is given by ε where $\varepsilon = x - X$

$$\text{Percentage error} = \frac{100\varepsilon}{x}$$

E.g. John drops a ball down a stairwell.
(1) He notes that it takes 2 seconds to hit the floor. Taking the time as exact and the value of $g = 9.8 \text{ m s}^{-2}$, find the height of the stairwell.

$$s = \frac{1}{2}gt^2$$

$$\Rightarrow s = \frac{1}{2} \times 9.8 \times 4 = 19.6 \text{ m}$$

(2) In fact, the time of 2 seconds is only correct to the nearest second. Find the upper and lower bounds for *s*.

$$\text{Maximum } s = \frac{1}{2}g(2.5)^2 = 30.625 \text{ m}$$

$$\text{Minimum } s = \frac{1}{2}g(1.5)^2 = 11.025 \text{ m}$$

N.B. given this variation in the timing of t note that the value of s found cannot even be given to 1 significant figure.

(3) John uses the experiment to find a value for *g*. He measures the depth of the drop to be 19.5 m correct to the nearest 0.1m and the time to the nearest second.
Find the error bounds for *g*.

$$g = \frac{2s}{t^2} \Rightarrow \text{Maximum } g = \frac{39.1}{1.5^2} = 17.4 \text{ (3 s. f.)}$$

$$\text{Minimum } g = \frac{38.9}{2.5^2} = 6.22 \text{ (3 s. f.)}$$

i.e. $6.22 < g < 17.4$

E.g. The length and breadth of a rectangular room are measured as 3 m by 4 m, each to the nearest 10 cm. Find the error bounds for the perimeter and area.

Greatest perimeter = 2(3.05 + 4.05) = 14.2 m
Least perimeter = 2(2.95 + 3.95)= 13.8 m
i.e. perimeter is 14m ± 20 cm

greatest area = 3.05 × 4.05 = 12.3525 m^2
Least area = 2.95 × 3.95 = 11.6525 m^2
i.e. $11.65\text{m}^2 < Area < 12.35 \text{ m}^2$ (3 s. f.)

E.g. Jane measures a stick to be 159 cm when the true length is 160 cm. What is the percentage error?

$$\text{Percentage error} = \frac{100(160-159)}{160} = 0.625\%$$